Avoiding Plagiarism in Mathematics

Richard D. Neidinger

Your English, history, and sociology professors talk a lot about how to write in those disciplines. They carefully define plagiarism and tell you about the best ways to avoid it. But as you progress in mathematics and begin to write mathematics papers, you may feel uncertain about how to appropriately use and cite mathematical work from outside sources.

Unlike in your humanities papers, you now need to use precise definitions, theorems, and proofs developed by others. Maybe your assignment is to explain ideas from one published paper. Are you allowed to copy? When must you reference?

The general advice that follows will hopefully help, whether you are writing for a homework assignment, a term paper, a senior thesis, a research project report, or a journal submission. Although the last requires more originality, all should express your thinking. Other media, such as research posters and slideshow presentations, should be held to similar standards.

Many documents on avoiding plagiarism recommend that you "look away" from a source while writing, but is this necessary in math? Briefly, for prose—yes; theorems—no; and proofs—to some extent. Now, on to explanations.

When to Look Away

In many respects, avoiding plagiarism in mathematics is much the same as in other subjects. The *MLA Handbook* instructs writers to "remember that you must document everything that you borrow—not only direct quotations and paraphrases but also information and ideas" [2, p. 55]. The purpose is to understand these sources and present your understanding, with ideas, reasoning, and certainly prose, in your own words.

When explaining a mathematical idea in a prose paragraph, you should *not* have the source open in front of you when writing. Take notes (from more than one source, if possible), and then write at least a paragraph, using your own sentences, without looking at the source. This is particularly challenging if writing in a second language, since it might be tempting to use sentence structure and grammatical patterns of your source [2, p. 55].

When to Look

It is often appropriate to copy precise statements such as definitions, theorems, claims, or concise algorithms, with no quotation marks. In such cases you may look directly at your source; paraphrasing is encouraged but not necessary. A reference is required unless the statement is well known and can be found in many sources.

Figures

You should create your own graphics or diagrams, usually labeled as figures, though instructors may allow a copied figure with reference for use within a course. For publication, reprinting a figure would require specific permission from the copyright holder.

Proofs

The gray area is proofs, derivations, examples, programs (extensive algorithms), or anything with mathematical or computational steps of logic and manipulation. We use the word "proof" below to stand in for any of these structures.

Consider three levels of personalizing the proof, usually reflecting levels of understanding:

- **Closed source.** In most cases, this is the recommended approach. Take notes from the source and work it out on your own paper in your personal style. When typing, look only at your notes and work. If possible, change an example, do a different case, or implement an algorithm using different software. Reference the source.
- **Open source.** Having a source open while writing (with reference) is acceptable for undergraduate papers, if you need it to follow the progression of the proof while writing. You must understand each step well enough to use personal wording and/or add more detailed explanation or comments to steps. In fact, math education research shows enhanced learning from such "self-explanation" of lines when-

ever you read a proof [1]. Check with your instructor if you are unsure about your level of exposition.

- **Copied source.** Copying is *unacceptable*. If the source makes sense to you (or not) but you don't understand it well enough to (or just don't want to) rephrase or explain steps, then it is best to simply write "the proof is found in [5]." If you must copy it (for example, to compare one approach to another), say explicitly that the proof is taken straight from the source.

References

References and citations are required but usually handled a little differently in mathematics. In the bibliography, which is usually called *References*, you list, exactly once, any source that helped your understanding of the material, no matter how many times (zero is allowed) it is cited. Online sources that are not peer-reviewed may help your understanding (and should be listed), but should not be used as primary sources.

Both the reference listing and the citations are usually a number, letter, or last name (or portion of a last name) inside square brackets (such as [5], [N], or [Nei]), not a footnote, and sometimes may contain specifics ([5, p. 361]). You can use the reference as a noun ("In [5] . . ."), or it can go before the period at the end of a sentence.

There is no standard format of the reference listing or citation style, although a particular professor or journal may have a required style. If not, pick a style and be consistent—for instance, use one of your sources as a model.

To Cite or Not to Cite?

In almost all circumstances—papers, articles, posters, and presentations—proper citation is required, but there are exceptions. For instance, previously covered course material probably can be used without citation (see instructor).

However, an untaught section of the course textbook should be referenced, particularly if the section is the topic of your exposition. Well-known material from the high school or calculus level, such as a trigonometric identity or the intermediate value theorem, may be used without reference, although this principle is relative to the level of your paper.

Unless you are writing about the history of mathematics, a theorem does not have to be traced to the original paper but can be referenced to a modern exposition, such as a textbook.

In research articles, theorems without a reference are claimed as original by the author, unless well known. In talks, presenters often claim original work with their initial and reference the work of others by name (a theorem of presenter Baker with collaborators Abel and Chu could be "Theorem [Abel, B, Chu]").

Some cultures might consider copying a person's work or style to be a way to honor that person, but our profession considers it plagiarism and a dishonor to that person. Terence Tao writes in [6], "If one wants to truly respect a mathematician, then understand that mathematician's methods, results, and exposition, and improve, update, adapt, and advance all three." This is how you learn, discover difficulties, and develop ideas, and it is how mathematics advances. ■

Further Reading

[1] L. Alcock, M. Hodds, S. Roy, M. Inglis, Investigating and improving Undergraduate Proof Comprehension, *Notices of the AMS* **62** (2015) 742-752.

[2] J. Gibaldi, *MLA Handbook for Writers of Research Papers*. Seventh edition. MLA, New York, 2009.

[3] N. J. Higham, *Handbook of Writing for the Mathematical Sciences*. Second edition. SIAM, Philadelphia, 1998.

[4] *Writing*, MAA Mathematical Communication website, *mathcomm.org/writing*.

[5] R. D. Neidinger, A fictional paper to provide an example to cite, *Never Published* **1** no. 1 (2016).

[6] T. Tao, Write in your own voice (retrieved July 16, 2015), *terrytao.wordpress.com/advice-on-writing-papers/write-in-your-own-voice*.

Richard D. Neidinger is professor of mathematics at Davidson College, where the student-implemented honor code is a cherished tradition. He enjoys and writes about analysis, numerical methods, and dynamical systems.
Email: rineidinger@davidson.edu

http://dx.doi.org/10.4169/mathhorizons.23.4.16