Math A68 – Quiz 3 – Tuesday 11/17/15 SOLUTIONS

Instructions: justify your answers where appropriate. Leave numerical answers unsimplified.

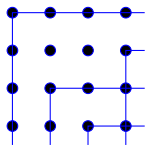
1. Give two permutation patterns of length 4 in the permutation $w = 7213564$.

Solution: 7213564: 4213, 7213564: 4213, 7213564: 4123, 7213564: 1342, ...

2. Give the dot diagram and the permutation matrix for the permutation $w = 1423$.

Solution:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



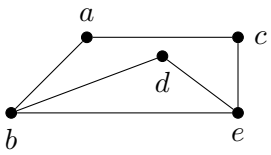
3. Explain why there exist no graphs, simple or otherwise, with degree sequence $4, 4, 3, 1, 1, 0, 0$.

Solution: By the handshake lemma, every graph has an even number of odd-degree vertices, but this sequence has an odd number of odd values.

4. Explain why, for a graph with at least three vertices, if every pair of vertices is contained in a common cycle, then the graph has vertex connectivity at least 2.

Solution: Let $u, v \in V$. Since u and v are contained in a common cycle, there are two internally disjoint paths P_1 and P_2 between them. So G is connected. Moreover, for any third vertex $w \in V$, either w is in P_1 or P_2 or neither, but not both, so u and v are also connected in $G - w$. So $\kappa(G) \geq 2$.

5. Consider

$G =$ 

(a) Give an example of a maximal length path in G.

Solution: For example, d, b, a, c, e.

(b) What is the edge connectivity number $\lambda(G)$?

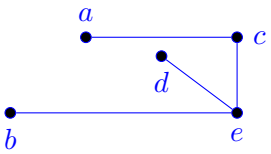
Solution: Since there are no cut-edges, but $\{ab, ce\}$ is an edge-cut, $\lambda(G) = 2$.

(c) Is G bipartite? Either show that it is or explain why it's not.

Solution: No, because it contains the odd cycle b, d, e, b.

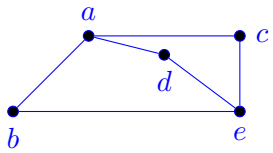
(d) Give an example of a spanning tree for G.

Solution:

$T =$ 

(e) Give an example of a graph which has the same degree sequence as G but is not isomorphic to G.

Solution:

$H =$ 

Both have the degree sequence $3, 3, 2, 2, 2$.