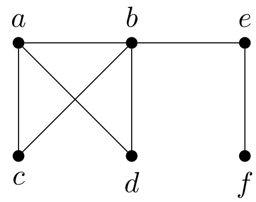
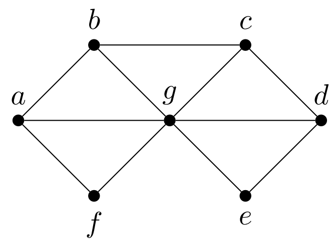
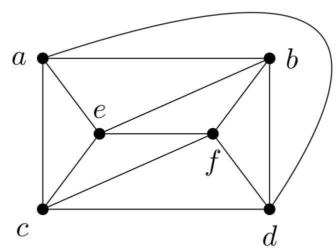
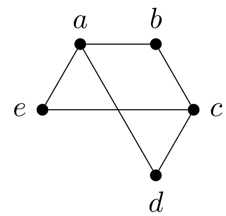
Exercise 36. Let A, B, C, and D be the graphs

$A =$ 

$B =$ 

$C =$ 

$D =$ 

(a) Calculate the chromatic numbers for A, B, C, and D. For each, give an example of a vertex coloring of the corresponding graph using exactly χ colors.

(b) What are the clique and independence numbers of A, B, C, and D? How do ω and $|V|/\alpha$ compare to χ for each graph?

(c) What are the chromatic numbers of

(i) $K_{m,n}$, (ii) C_n, (iii) W_n, and (iv) Q_n?

(d) What are the clique and independence numbers of

(i) $K_{m,n}$, (ii) C_n, (iii) W_n, (iv) Q_n?

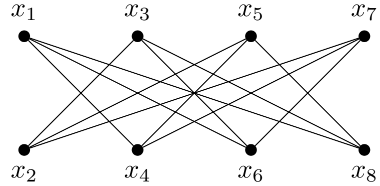
How do ω and $|V|/\alpha$ compare to χ for each graph? (You may need to break into cases.)

(e) What are necessary and sufficient conditions for a graph to have chromatic number (i) 1, and (ii) 2.

(f) For A, B, D, and H, pick an ordering of the vertices and color the graph using the greedy algorithm. Compare the colors you used to the chromatic number of the graph. Can you find an ordering that yields a coloring with too many colors?

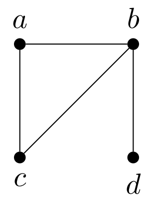
(g) Explain why the clique number of the complement of a bipartite is no smaller than the number of vertices in each part. (Recall that the *parts* of a bipartite graph are the two collections of pairwise non-adjacent vertices.)

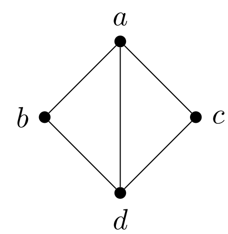
(h) Notice that the graph

$G =$ 

is bipartite, so should have chromatic number 2. Now color this graph using the greedy algorithm. How many colors did you need?

Exercise 37. (a) Compute the number of ways to color the graph with each of 1, 2, ..., $|V|$ colors for

$G =$ 

and $H =$ 

(b) Calculate the chromatic polynomial for G above.

(c) The chromatic polynomial for the cycle C_n is $\chi(C_n, k) = (k-1)^n + (-1)^n(k-1)$.

(i) Draw all the ways of coloring the 3-cycle with 3 colors. Then compute $\chi(C_3, 3)$ and compare your answers.

(ii) How many ways are there to color the 5-cycle with 3 colors?

(iii) How many ways are there to color the 6-cycle with 2 colors?

(iv) Use $\chi(C_n, k)$ to verify that even cycles are bipartite and odd cycles are not.