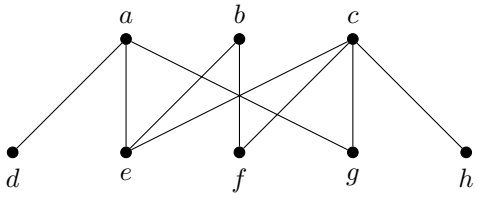
Exercise 32. Complete the proof of the following theorem:
The following are equivalent for a simple graph G.

(1) G is a tree.
(2) G is a *minimal connected graph*, i.e. every edge in E is a cut edge.
(3) G is a maximally acyclic graph, i.e. G is acyclic, and adding any edge between two non-adjacent vertices creates a cycle.

Exercise 33. (a) How many spanning trees does C_5 have?

(b) Let

$G =$ 

(a) Calculate $\text{diam}(G)$ and $\text{rad}(G)$.
(b) Fix the vertex a, and give $V_i = \{u \in V \mid d(u, a) = i\}$.
(c) Build a spanning tree using Method 1 from the notes using $v = a$ (show your steps!). What is the radius of the resulting tree?
(d) Find a central vertex v, i.e. one for which $\max_{u \in V} d(u, v) = \text{rad}(G)$, and build a spanning tree using Method 1 from the notes using that vertex (show your steps!). What is the radius of the resulting tree?
(e) Build a spanning tree using Method 2 from the notes starting with T_1 being the isolated vertex a (show your steps!).