Combinatorial Analysis – 11/05/15

Exercise 31.

(a) Draw the isomorphism classes of connected graphs on 4 vertices, and give the vertex and edge connectivity number for each.

(b) Show that if v is a vertex of odd degree, then there is a path from v to another vertex of odd degree.

(c) Prove that for every simple graph, either G is connected, or $\bar{G}$ is connected.

(d) Recall that $\kappa(G)$ is the vertex connectivity of G and $\lambda(G)$ is the edge connectivity of G. Give examples of graphs for which each of the following are satisfied.

- (i) $\kappa(G) = \lambda(G) < \min_{v \in V} \deg(v)$
- (ii) $\kappa(G) < \lambda(G) = \min_{v \in V} \deg(v)$
- (iii) $\kappa(G) < \lambda(G) < \min_{v \in V} \deg(v)$
- (iv) $\kappa(G) = \lambda(G) = \min_{v \in V} \deg(v)$

(e) For the following theorem, pick any of parts (ii)–(iv) and show (carefully!) that it's equivalent to part (i).

Theorem: For a simple graph with at least 3 vertices, the following are equivalent.

- (i) G is connected and contains no cut vertex.
- (ii) Every two vertices in V are contained in some cycle.
- (iii) Every two edges in E are contained in some cycle, and G contains no isolated vertices.
- (iv) For any three vertices $u, v, w \in V$, there is a path from u to v containing w.