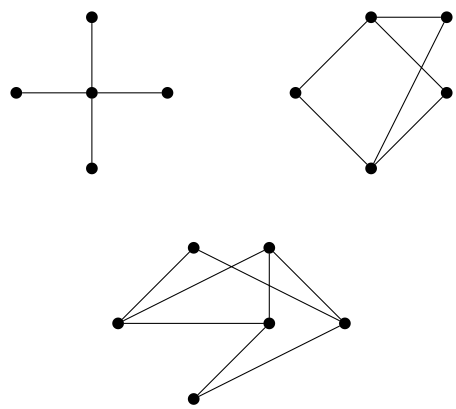
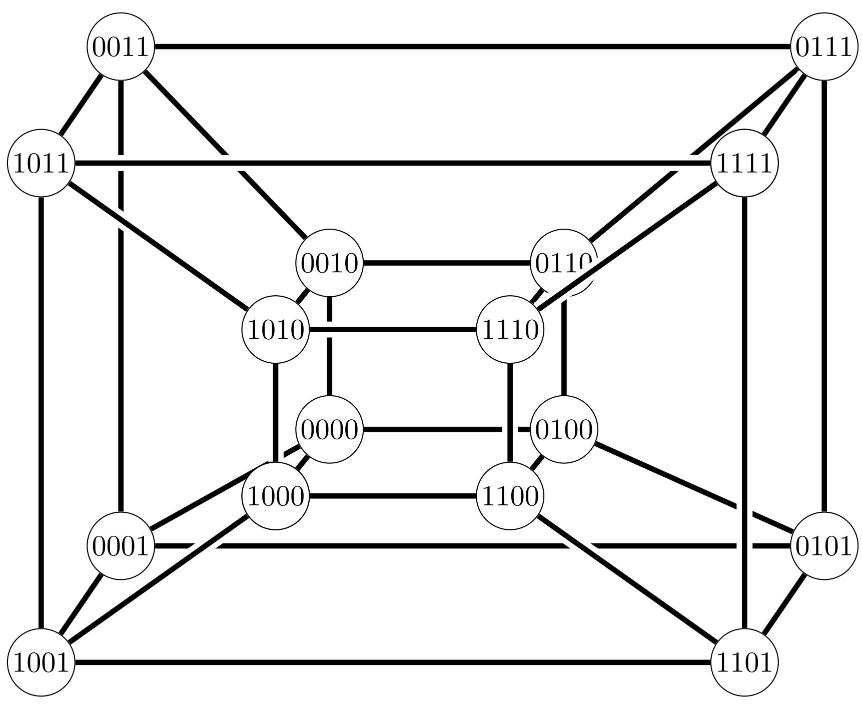
Exercise 28. (a) Draw C_6, W_6 K_6, and $K_{5,3}$.
(b) Which of the following are bipartite? Justify your answer.



(c) Hypercubes are bipartite.
(i) The following is the 4-cube:

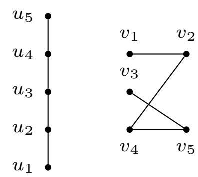


Shade in the vertices that have an even number of 0's. Explain why the 4-cube is bipartite.
(ii) Explain why Q_n is bipartite in general.
[Hint: consider the parity of the number of 0's in the label of a vertex.]

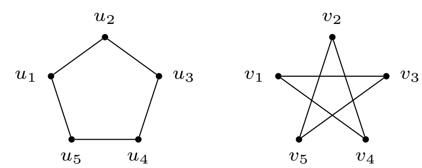
Exercise 29.

(a) For each of the following pairs of graphs, first list their degree sequences. Then decide whether they are isomorphic or not. If not, say why. If they are, give a bijection on the vertices that preserves the edges, and draw the unlabeled graph that represents the corresponding isomorphism class of graphs.

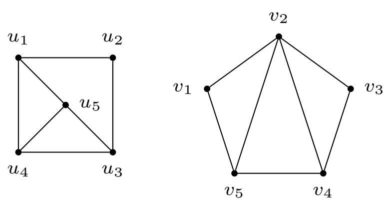
(i)



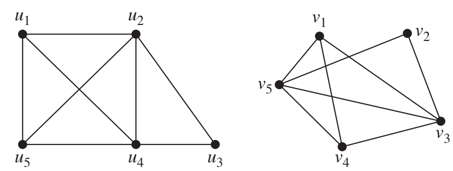
(ii)



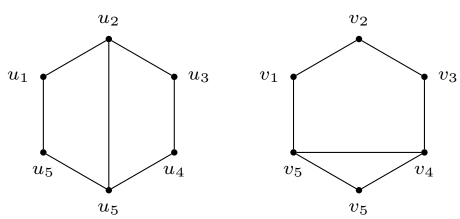
(iii)



(iv)



(v)



(b) How many isomorphism classes are there for graphs with 4 vertices? Draw them.

(c) How many edges does a graph have if its degree sequence is $4, 3, 3, 2, 2$? Draw a graph with this degree sequence. Can you draw a simple graph with this sequence?

(d) For which values of n, m are these graphs regular? What is the degree?

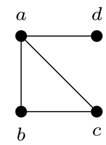
(i) K_n (ii) C_n (iii) W_n (iv) Q_n (v) $K_{m,n}$

(e) How many vertices does a regular graph of degree four with 10 edges have?

(f) Show that every non-increasing finite sequence of nonnegative integers whose terms sum to an even number is the degree sequence of a graph (where loops are allowed). Illustrate your proof on the degree sequence 7,7,6,4,3,2,2,1,0,0. [Hint: Add loops first.]

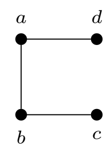
(g) Show that isomorphism of simple graphs is an equivalence relation.

Exercise 30. (a) Consider the graph

$G =$ 

(i) Give an example of a subgraph of G that is not induced.
(ii) How many induced subgraphs does G have? List them.
(iii) How many subgraphs does G have?
(iv) Let e be the edge connecting a and d. Draw $G - e$ and G/e.
(v) Let e be the edge connecting a and c. Draw $G - e$ and G/e.
(vi) Let e be an edge connecting d and c. Draw $G + e$.
(vii) Draw $\bar{G}$.

(b) Show that

$G =$ 

is isomorphic to its complement.

(c) Find a simple graph with 5 vertices that is isomorphic to its own complement. (Start with: how many edges must it have?)