Exercise 23. Inversions and descents.

(a) For each of $w \in \mathcal{S}_3$, write w in word form and give (i) w^{-1}, (ii) $I(w)$, (iii) $\mathrm{inv}(w)$, (iv) $\mathrm{code}(w)$, (v) $D(w)$, (vi) $\mathrm{des}(w)$, and (vii) $\mathrm{maj}(w)$. (Make a table.)

(b) Use your calculations in (a) to verify
   (i) $I(w^{-1})$ and $\mathrm{code}_i(w) = \#\{j > i \mid w(j) < w(i)\}$ are equivalent definitions of $\mathrm{code}(w)$,
   (ii) Corollary 1.3.13,
   (iii) Proposition 1.3.14,
   (iv) the proof of Prop 1.3.14 (show the bijection between inversions (i, j) in w and inversions (w_i, w_j) in w^{-1}); and
   (v) equation (1.41),

   for $n = 3$.