Welcome to Combinatorial Analysis!

Warmup. Count the following.

(a) The number of possible outcomes of flipping a coin 3 times.

(b) The number of possible outcomes of flipping a coin 10 times.

(c) The number of possible outcomes of flipping a coin 10 times so that the first flip comes up tails, but the last flip doesn't.

(d) The number of possible outcomes of flipping a coin 10 times so that either the first flip comes up tails or the last flip comes up tails, but not both.

(e) The number of ways to pick a president, a vice president, and a secretary for a club with 10 members.

(f) The number of ways to separate four distinct playing cards into two sets of two.

Course logistics:

Professor: Zajj Daugherty, NAC 6/301, zdaugherty@ccny.cuny.edu

Office hours: (For now) Tu 3:45–4:45, Th 11–12, or by appointment.

Course webpage: http://zdaugherty.ccnysites.cuny.edu/teaching/mA6800f15/

Textbook: Enumerative Combinatorics, volume 1 (2nd edition), by Richard Stanley. (Online!)

Grades: 20% homework and quizzes, 35% Midterm, 45% Final.
Homework due on Thursdays in class, assigned online. Exam/quiz dates TBA.

Homework 0: Due Monday 8/31.

Email zdaugherty@ccny.cuny.edu from the email address at which you would like me to contact you. Include the following.

(a) What name you like to go by, and how is it pronounced.

(b) Why you're in this class/program and how it pertains to your goals (be specific).

(c) Something interesting about yourself.

(d) Optional: A photo or a description of yourself to help me put your name to a face faster.

(e) Whether or not you would be interested in exchanging information with classmates for study-group purposes. If so, include a phone number. I will send out an email next week to everyone who opts in with names, emails, and phone numbers.

Exercise 1. Extended warmup.

(a) Write out the following sets explicitly.

(i) $[4]^2$ (ii) $2^{[4]}$ (iii) $S = \{(a,b) \mid a \in [2,4], b \in [-4,7]\}$ (iv) $[4]^2 \cap 2^{[4]}$ (v) $[4]^2 \cap S$ (vi) $2^{[3]} \cup 2^{[4]}$

(b) For sets A and B, decide whether the following identities are **true or false**, and why.

(i) $A \cap B = B \cap A$ (ii) $A \cup B = B \cup A$ (iii) $A - B = B - A$ (iv) $|A - B| = |A| - |B|$

(c) Answer the following counting problems, leaving your numerical answer unsimplified.

(i) A particular kind of shirt comes in two different cuts–male and female, each in three color choices and five sizes. How many different choices are made available?

(ii) On a 10-question true-or-false quiz, how many different ways can a student fill out the quiz if they answer all of the questions? if they might leave questions blank?

(iii) How many 3-letter words (not "real" words, just strings of letters) are there?

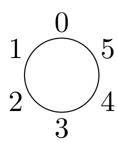
(iv) How many 3-letter words are there that have no repeated characters?

(v) How many 3-letter words are there that have the property that if they start in a vowel then they don't end in a vowel?

Exercise 2 (EC 1.2)**.** Give as simple a solution as possible. Justify your answers (using words).

(a) How many subsets of the set $[10] = \{1, 2, \ldots, 10\}$ contain at least one odd integer?

(b) In how many ways can six people be seated in a circle if two seatings are considered the same whenever each person has the same neighbors (not necessarily on the same side)? For example,



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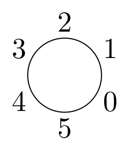
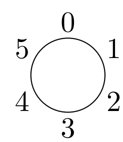
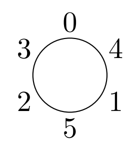


and


but not


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(c) A *permutation* of a finite set S is a bijective map $w : S \to [n]$, where $n = |S|$.

(i) How many permutations $w : [6] \to [6]$ are there?

(ii) How many permutations $w : [6] \to [6]$ satisfy $w(1) \neq 2$?

A *cycle* of a permutation is a sequence $(c_1, c_2, \ldots, c_\ell)$ such that

$$w : c_1 \mapsto c_2, \quad w : c_2 \mapsto c_3, \quad \ldots \quad w : c_{\ell-1} \mapsto c_\ell, \quad w : c_\ell \mapsto c_1.$$

For example, the permutation $w : [4] \to [4]$ given by

$$1 \mapsto 4, \quad 2 \mapsto 2, \quad 3 \mapsto 1, \quad 4 \mapsto 3$$

has exactly two cycles: $(1,4,3)$ and (2).

(iii) How many permutations $w : [6] \to [6]$ have exactly one cycle? [Hint: question (b) $\times 2$.)

(iv) How many permutations $w : [6] \to [6]$ have exactly two cycles of length 3?

(d) There are four people who want to sit down, and six distinct chairs in which to do so. In how many ways can this be done?

Exercise 3. (a) Explain why $\binom{n}{k} = \frac{(n)_k}{k!}$ directly using product and division rules.

(b) Give a combinatorial proof of the identity

$$\sum_{i=0}^{n} \binom{a}{i}\binom{b}{n-i} = \binom{a+b}{n},$$

where $a, b, n \in \mathbb{N}$ and $a, b \geq n$. [Hint: Consider two disjoint sets A and B, with $|A| = a$ and $|B| = b$. How many subsets does $A \sqcup B$ have?]