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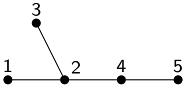
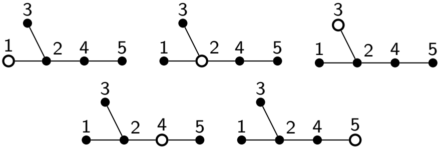


then there are 5 associated *rooted* trees:



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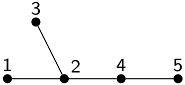

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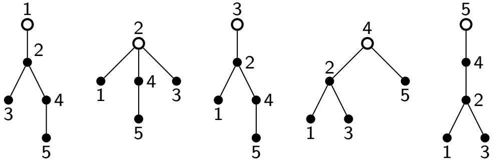

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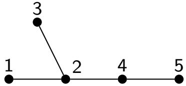

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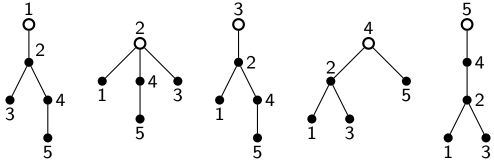

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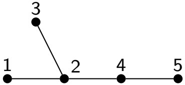

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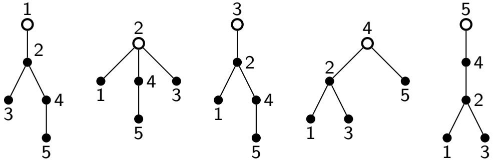

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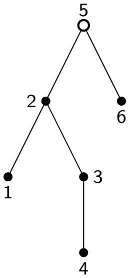

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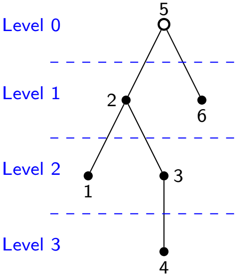

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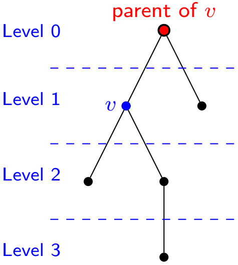

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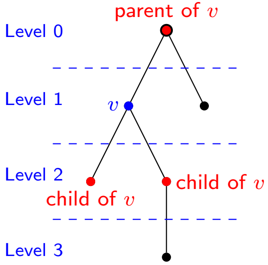

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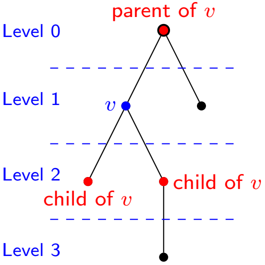

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Recall a leaf is a vertex of degree 1; note that leaves are the vertices with no children. Everything else is called an internal vertex.

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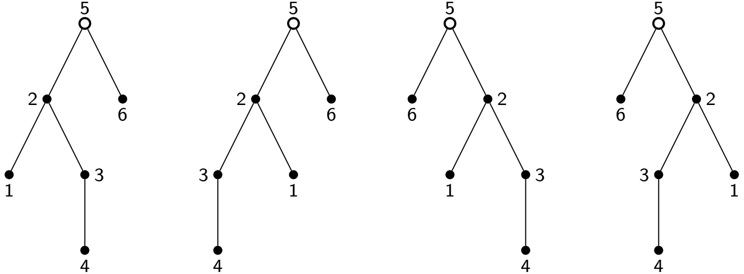

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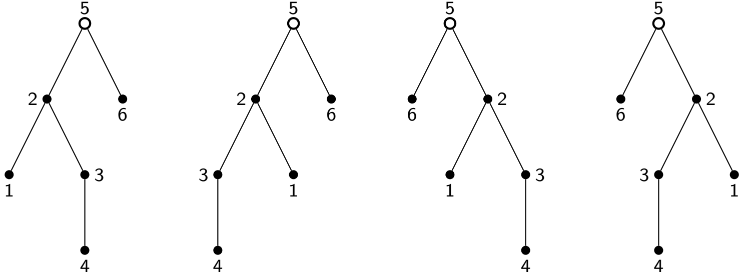

Given a rooted tree, there are $\boxed{\prod_{v \text{ internal}} (\deg(v)-1)}$ associated ORT's.

You try: How many ORT's are there on the vertex set $V = \{1, 2, 3, 4\}$?

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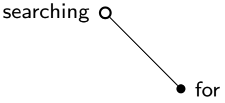

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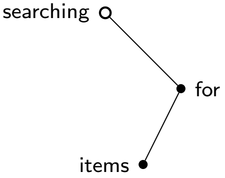

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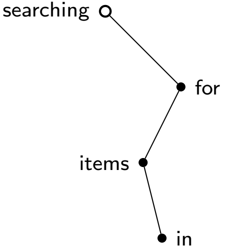

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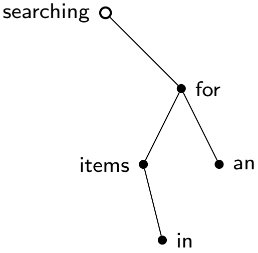

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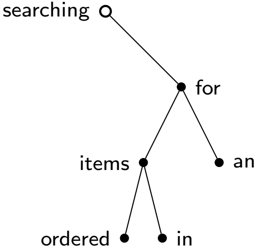

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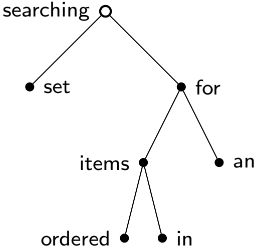

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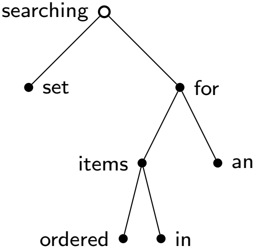

You try: add the words { build, the, search, tree } to the above ORT.

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Remark: There are algorithms for balancing search trees as they get built. (See: "data structures")

Moral: Building the tree takes some work, but once it's built, it reduces the computational complexity of finding items.

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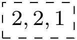

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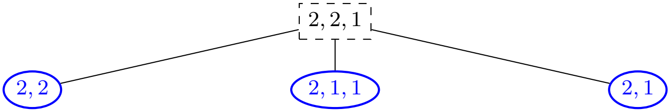

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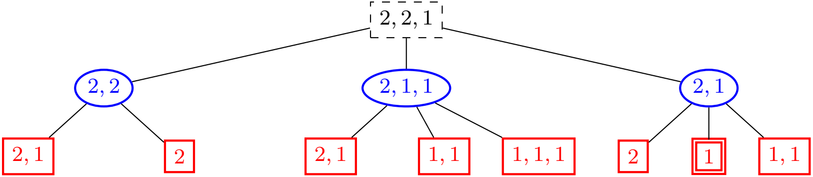

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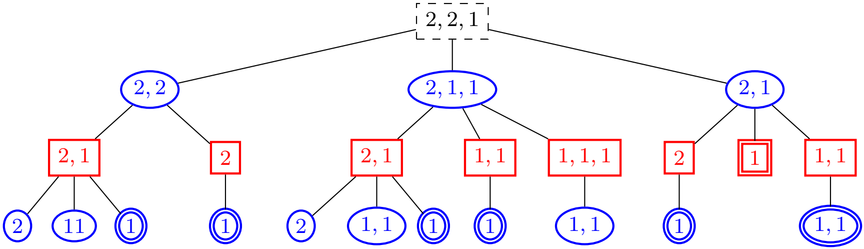

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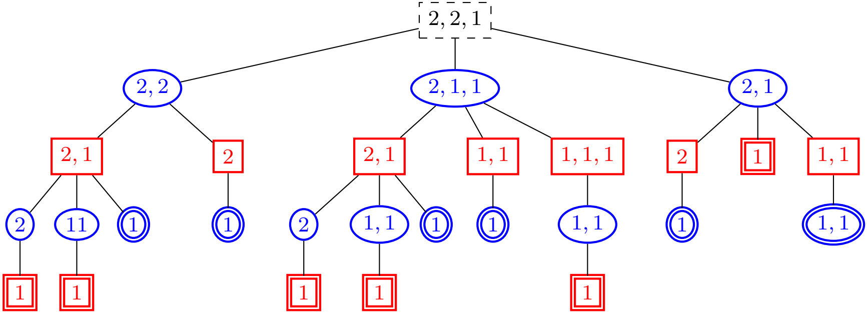

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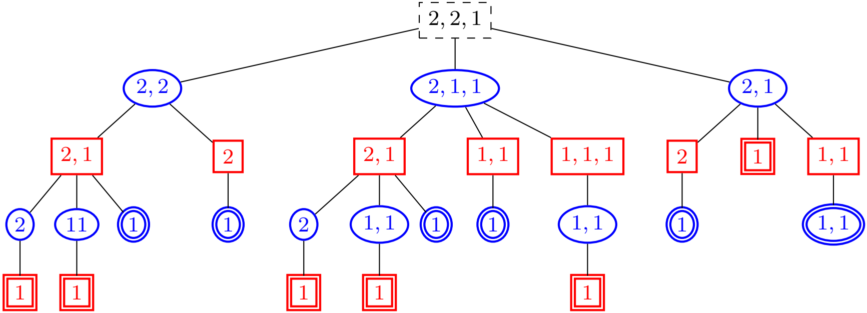

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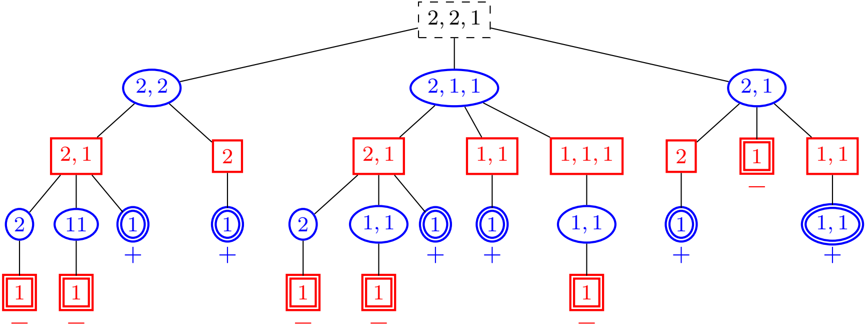

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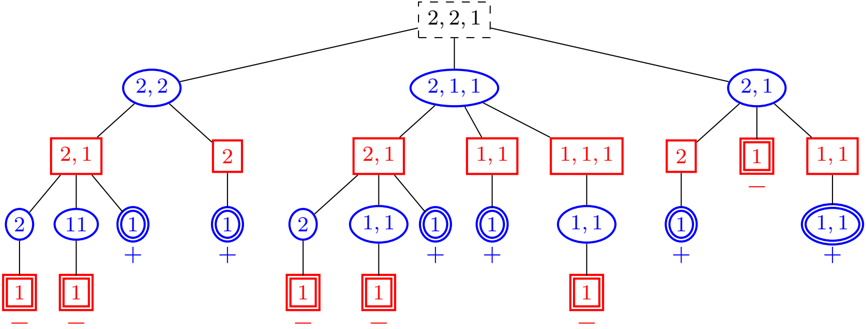

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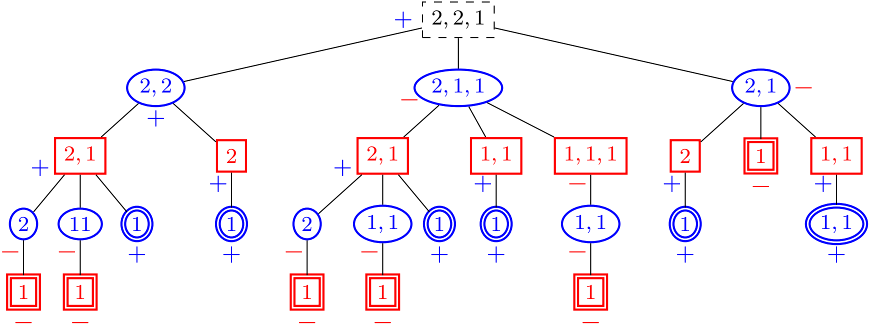
Nim: 2 players start with piles of stones. Taking turns, each player takes one or more stones from any one pile. The player to take the last stone loses. Example: Start with 2, 2, and 1. The associated game tree is...

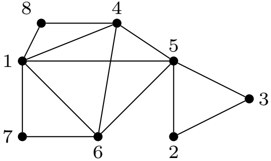


Next, assign +1 to leaves where player 1 wins, and -1 to leaves where player 2 wins. For internal vertices: give player 1 the min value of its children, give player 2 (and the root) the max value of its children.

Thm: The value says who will win if each player follows a min/max strategy.

Application 3: Searching graphs

Let G be a simple connected graph, and put an order on the vertices.

Ex: $G =$ 

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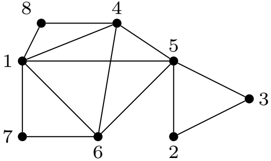
Ex: $G =$ 

Depth-first search:

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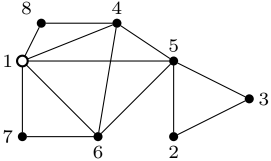
Ex: $G =$ 

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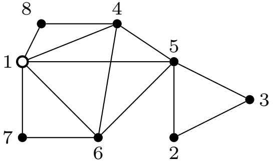
Ex: $G =$ 

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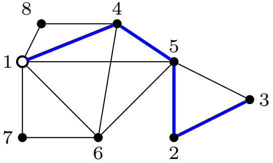
Ex: $G =$ 

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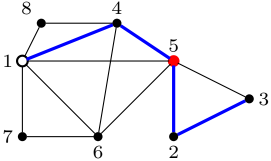
Ex: $G =$ 

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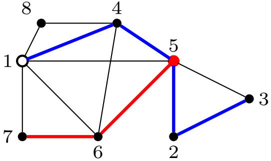
Ex: $G =$ 

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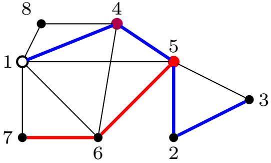
Ex: $G =$ 

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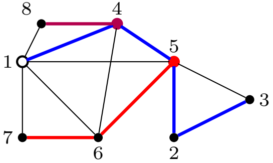
Ex: $G =$ 

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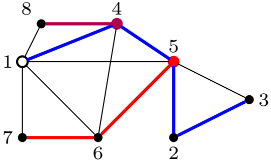
Ex: $G =$ 

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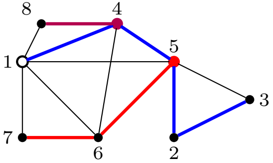
Ex: $G =$ 

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The result is a *rooted spanning tree*.

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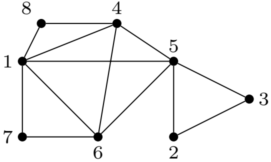
Ex: $G =$ 

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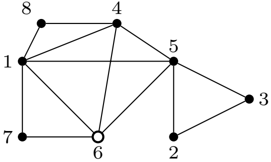
Ex: $G =$ 

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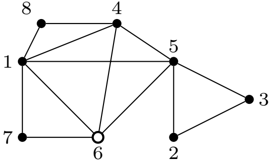
Ex: $G =$ 

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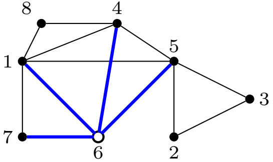
Ex: $G =$ 

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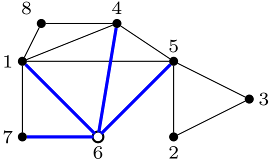
Ex: $G =$ 

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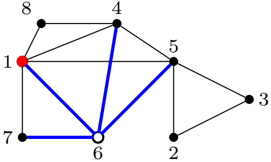
Ex: $G =$ 

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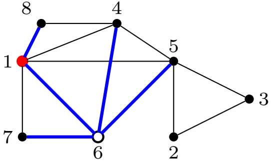
Ex: $G =$ 

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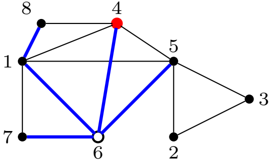
Ex: $G =$ 

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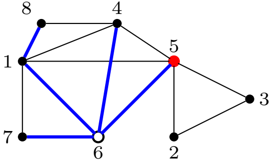
Ex: $G =$ 

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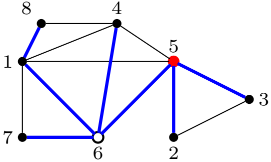

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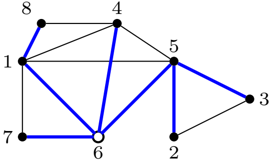
Ex: $G =$ 

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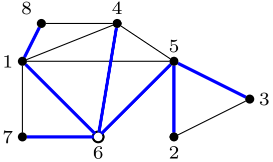
Ex: $G =$ 

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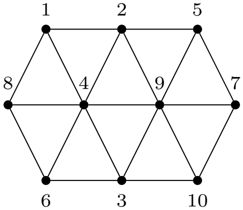
The result is also a *rooted spanning tree*. Note that at each recursion, you're building all of the vertices at a given level.

You try: Starting from vertex 1, compute the depth-first and breadth-first search trees for the graph



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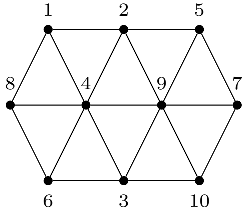
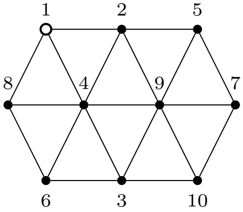
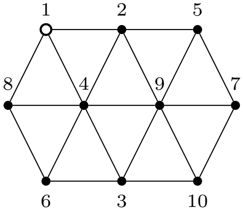


Depth:



Breadth:



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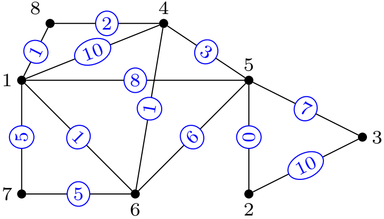

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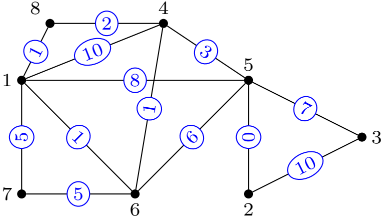

Relevant questions:

- Given vertices u and v, what's the smallest-weight walk from u to v? (Think: flights, cab rides, production lines, etc.)
- What is the smallest-weight spanning tree?

See also: Traveling salesman problem.

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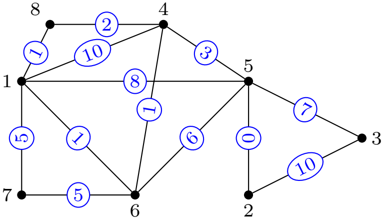

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1. Pick an edge of minumum weight and add it (and its vertices) to your tree.
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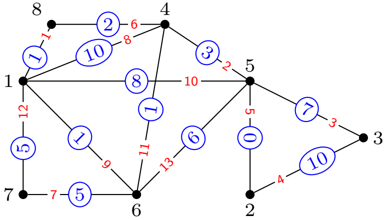

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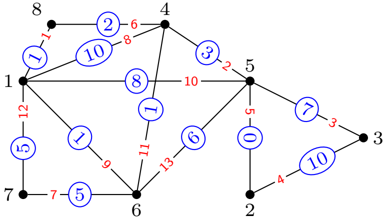

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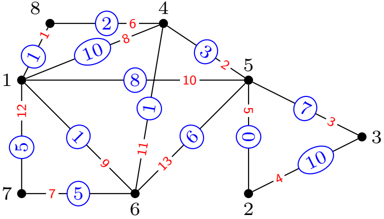

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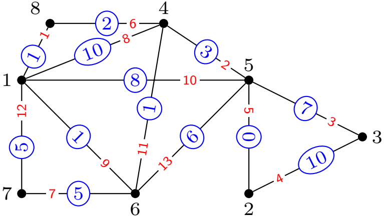

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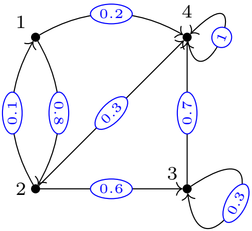
Ex: $G =$ 

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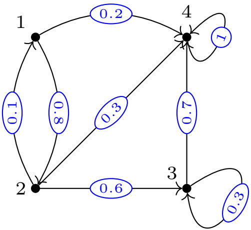
Ex: $G =$ 

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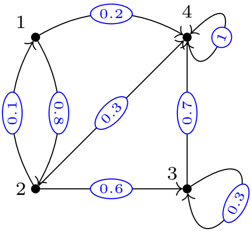

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Example: Suppose you play a game of dice, where at each turn you roll two six-sided dice. If you roll a multiple of 4, you get \$3; if you don't, you pay \$1.

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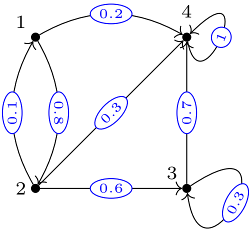

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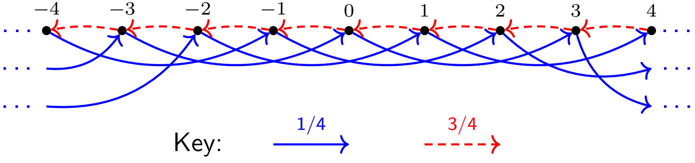

Relevant questions: Starting at vertex u. . .

1. What's the probability that you'll reach vertex v?
2. After n steps, what's the probability that you've landed at v?
3. Is it more likely for a walk gravitate toward any one vertex? Is it more likely that a random walk wanders off in any particular direction?