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For two sets A and B, we have

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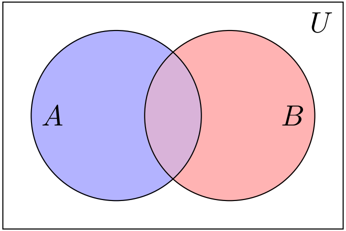

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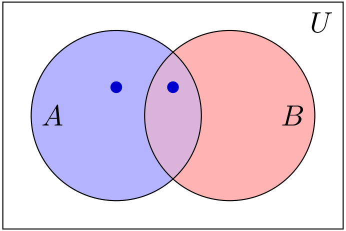

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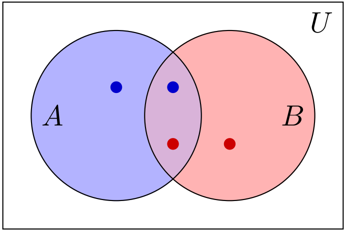

Inclusion/exclusion

For three sets A, B, and C...

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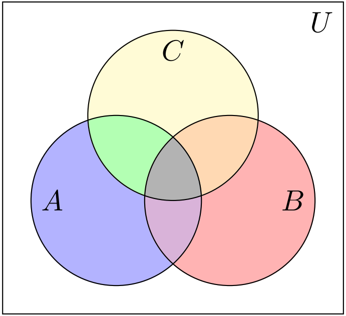

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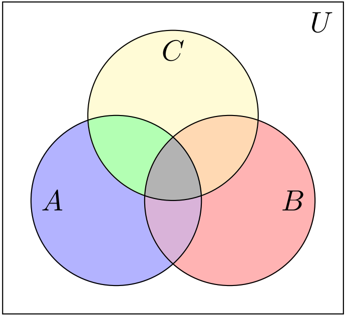

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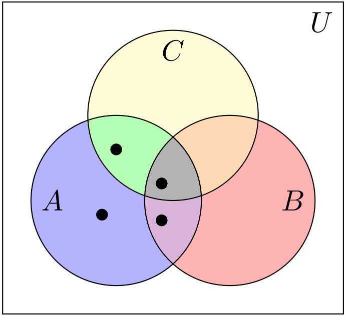

$$|A \cup B \cup C| = |A|$$

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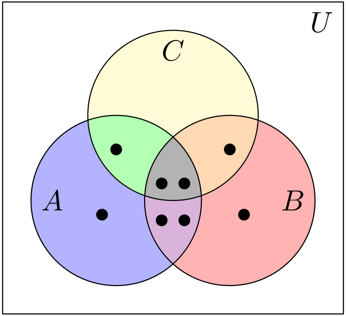

$$|A \cup B \cup C| = |A| + |B|$$

.

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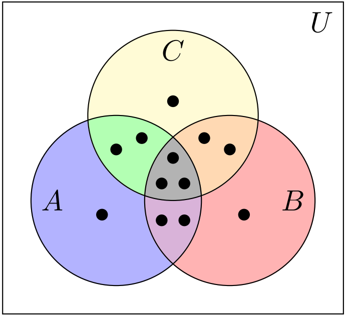

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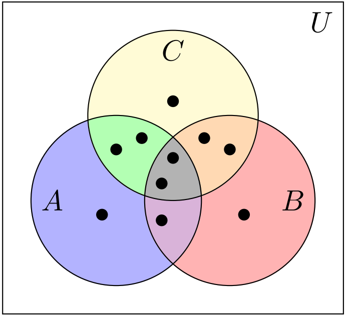

$$|A \cup B \cup C| = |A| + |B| + |C|$$
$$- (|A \cap B| \qquad)$$
$$.$$

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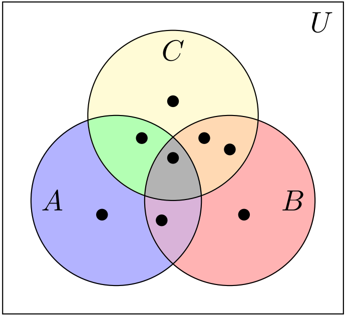

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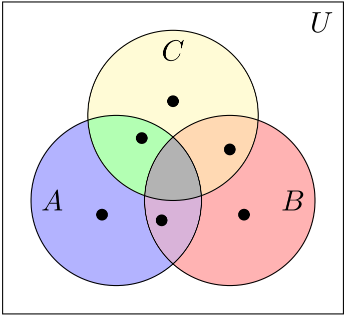

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|)$$

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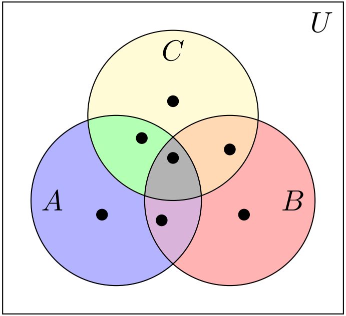

$$
\begin{aligned}
|A \cup B \cup C| = {} & |A| + |B| + |C| \\
& - (|A \cap B| + |A \cap C| + |B \cap C|) \\
& + |A \cap B \cap C|.
\end{aligned}
$$

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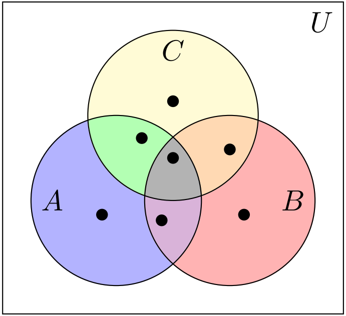

$$\begin{aligned} |A \cup B \cup C| = {} & |A| + |B| + |C| && \text{“include”} \\ & - (|A \cap B| + |A \cap C| + |B \cap C|) && \text{“exclude”} \\ & + |A \cap B \cap C|. && \text{“include”} \end{aligned}$$

Example: How many integers are there $1 \leqslant n \leqslant 100$ that are multiples of 2, 3, and/or 5?

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$|A \cap B| = \lfloor 100/(2 * 3) \rfloor = 16$
$|A \cap C| =$
$|B \cap C| =$

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Fact: n being a multiple of 2 and 3 is the same as being a multiple of 6. Same for 2 and 5 versus 10, 3 and 5 versus 15, and 2, 3, and 5 versus 30.

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So

$$|A \cup B \cup C| = 50 + 33 + 20 - 16 - 10 - 6 + 3 = \boxed{74}.$$

Inclusion/exclusion

Thm. For sets A_1, A_2, ..., A_n, we have

$$|A_1 \cup \cdots \cup A_n| = \sum_{S \subseteq \{A_1, \ldots, A_n\}} (-1)^{|S|-1} \left| \bigcap_{A_i \in S} A_i \right|$$

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Process this statement for $n = 3$:

Start with sets A_1, A_2, and A_3...

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S	$\varnothing$	$\{A_1\}$	$\{A_2\}$	$\{A_3\}$

S	$\{A_1, A_2\}$	$\{A_1, A_3\}$	$\{A_2, A_3\}$	$\{A_1, A_2, A_3\}$

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S	$\varnothing$	$\{A_1\}$	$\{A_2\}$	$\{A_3\}$
$\|S\|$	0	1	1	1

S	$\{A_1, A_2\}$	$\{A_1, A_3\}$	$\{A_2, A_3\}$	$\{A_1, A_2, A_3\}$
$\|S\|$	2	2	2	3

Inclusion/exclusion

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S	$\varnothing$	$\{A_1\}$	$\{A_2\}$	$\{A_3\}$
$\|S\|$	0	1	1	1
$(-1)^{\|S\|-1}$	-1	1	1	1

S	$\{A_1, A_2\}$	$\{A_1, A_3\}$	$\{A_2, A_3\}$	$\{A_1, A_2, A_3\}$
$\|S\|$	2	2	2	3
$(-1)^{\|S\|-1}$	-1	-1	-1	1

Inclusion/exclusion

Thm. For sets $A_1, A_2, \ldots, A_n$, we have

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S	$\varnothing$	$\{A_1\}$	$\{A_2\}$	$\{A_3\}$
$\|S\|$	0	1	1	1
$(-1)^{\|S\|-1}$	-1	1	1	1
$\bigcap_{A_i \in S} A_i$	$\varnothing$	A_1	A_2	A_3

S	$\{A_1, A_2\}$	$\{A_1, A_3\}$	$\{A_2, A_3\}$	$\{A_1, A_2, A_3\}$
$\|S\|$	2	2	2	3
$(-1)^{\|S\|-1}$	-1	-1	-1	1
$\bigcap_{A_i \in S} A_i$	$A_1 \cap A_2$	$A_1 \cap A_3$	$A_2 \cap A_3$	$A_1 \cap A_2 \cap A_3$

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Start with sets A_1, A_2, and A_3...

Try Exercises 41 & 42

S	$\varnothing$	$\{A_1\}$	$\{A_2\}$	$\{A_3\}$
$\lvert S\rvert$	0	1	1	1
$(-1)^{\lvert S\rvert-1}$	-1	1	1	1
$\bigcap_{A_i \in S} A_i$	$\varnothing$	A_1	A_2	A_3

S	$\{A_1, A_2\}$	$\{A_1, A_3\}$	$\{A_2, A_3\}$	$\{A_1, A_2, A_3\}$
$\lvert S\rvert$	2	2	2	3
$(-1)^{\lvert S\rvert-1}$	-1	-1	-1	1
$\bigcap_{A_i \in S} A_i$	$A_1 \cap A_2$	$A_1 \cap A_3$	$A_2 \cap A_3$	$A_1 \cap A_2 \cap A_3$

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Let $\sim$ be the relation on $\mathbb{Z}$ given by $a \sim b$ whenever $a < b$.

Example: $A = \mathbb{R}$ and $R = \{a \sim b \mid a = b\}$.

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Example: $A = \mathbb{R}$ and $R = \{a \sim b \mid a = b\}$.
In words:

Let $\sim$ be the relation on $\mathbb{R}$ given by $a \sim b$ whenever $a = b$.

Relations

A binary relation on a set A is a subset $R \subseteq A \times A$, where elements (a, b) are written as $a \sim b$.

Example: $A = \mathbb{Z}$ and $R = \{a \sim b \mid a < b\}$.
In words:

Let $\sim$ be the relation on $\mathbb{Z}$ given by $a \sim b$ whenever $a < b$.

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In words:

Let $\sim$ be the relation on $\mathbb{R}$ given by $a \sim b$ whenever $a = b$.

Example: $A = \mathbb{Z}$ and

$R = \{a \sim b \mid a$ and b have the same remainder when divided by 3$\}$.

More examples of (binary) relations:

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1. For A a number system, let $a \sim b$ if $a = b$.
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A binary relation on a set A is...

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1. For A a number system, let $a \sim b$ if $a = b$. R
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$$(a \sim b \wedge b \sim c) \Rightarrow a \sim c$$

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An equivalence relation on a set A is a binary relation that is reflexive, symmetric, *and* transitive.

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An equivalence relation on a set A is a binary relation that is reflexive, symmetric, *and* transitive. (Only #1)

Fix $n \in \mathbb{Z}_{>0}$ and define the relation on $\mathbb{Z}$ given by

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Is $\sim$ is an equivalence relation?

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Check: we have $a \sim b$ whenever $a - b = kn$ for some $k \in \mathbb{Z}$.

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reflexivity:

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reflexivity: $a - a = 0 = 0 \cdot n$ ✓

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integer:	-3	-2	-1	0	1	2	3	4	5	6	7	8
remainder:	2	3	4	0	1	2	3	4	0	1	2	3

So $0 \sim 5$, and $-2 \sim 3 \sim 8$, but $-3 \not\sim 3$.

Check: we have $a \sim b$ whenever $a - b = kn$ for some $k \in \mathbb{Z}$.

reflexivity: $a - a = 0 = 0 \cdot n$ ✓

symmetry: If $a - b = kn$, then $b - a = -kn = (-k)n$. ✓

transitivity: If $a - b = kn$ and $b - c = \ell n$, then

$$a - c = (a - b) + (b - c) = kn + \ell n = (k + \ell)n. ✓$$

Yes! This is an equivalence relation!

Let A be a set. Consider the relation on $\mathcal{P}(A)$ by

$$S \sim T \qquad \text{if} \qquad S \subseteq T$$

Is $\sim$ is an equivalence relation?

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Is

$$S \sim T \qquad \text{if} \qquad |S| = |T|$$

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Let $\sim$ be an equivalence relation on a set A, and let $a \in A$. The set of all elements $b \in A$ such that $a \sim b$ is called the equivalence class of a, denoted by $[a]$.

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Example: Consider the equivalence relation on $A = \{a, b, c\}$ given by

$$a \sim a, \quad b \sim b, \quad c \sim c, \quad a \sim c, \quad \text{and} \quad c \sim a.$$

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Then

$$[a] = \{a, c\} = [c], \quad \text{and}$$

$$[b] = \{b\}$$

are the two equivalence classes in A (with respect to this relation).

(We say there are two, *not three*, since "the equivalence classes" refers to the sets themselves, not to the elements that generate them.)

Let $\sim$ be an equivalence relation on a set A, and let $a \in A$. The set of all elements $b \in A$ such that $a \sim b$ is called the equivalence class of a, denoted by $[a]$.

Example: We showed that

$$\text{“}a \sim b \quad \text{if } a - b = 5k \text{ for some } k \in \mathbb{Z}\text{”}$$

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$$A = \bigsqcup_{a \in A} [a], \qquad \text{LaTeX: \textbackslash bigsqcup, \textbackslash sqcup}$$

For example, in our last example, there are exactly 5 equivalence classes: $[0]$, $[1]$, $[2]$, $[3]$, and $[4]$.

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$$A = \bigsqcup_{a \in A} [a], \qquad \text{LaTeX: \textbackslash bigsqcup, \textbackslash sqcup}$$

For example, in our last example, there are exactly 5 equivalence classes: $[0]$, $[1]$, $[2]$, $[3]$, and $[4]$. Any other seemingly different class is actually one of these (for example, $[5] = [0]$).

Theorem. The equivalence classes of A partition A into subsets, meaning

1. the equivalence classes are subsets of A:
$$[a] \subseteq A \text{ for all } a \in A;$$
2. any two equivalence classes are either equal or disjoint:
$$\text{for all } a, b \in A, \text{ either } [a] = [b] \text{ or } [a] \cap [b] = \varnothing; \text{ and}$$
3. the union of all the equivalence classes is all of A:
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So $\boxed{\mathbb{Z} = [0] \sqcup [1] \sqcup [2] \sqcup [3] \sqcup [4]}$.