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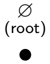

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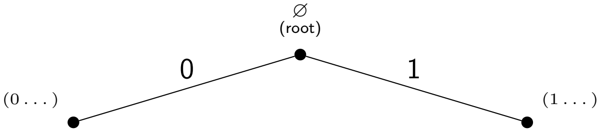

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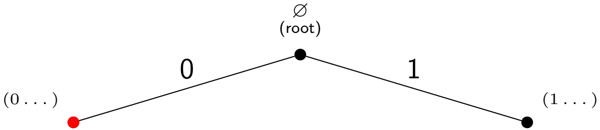

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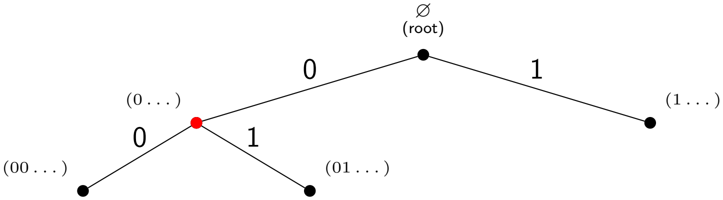

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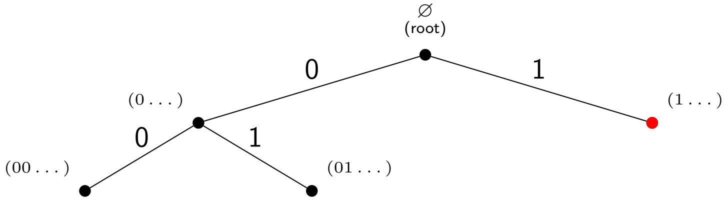

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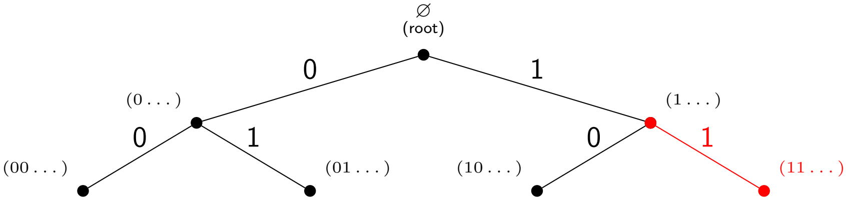

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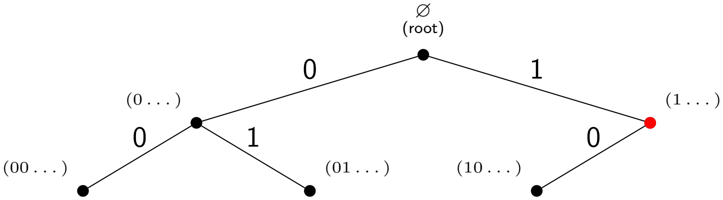

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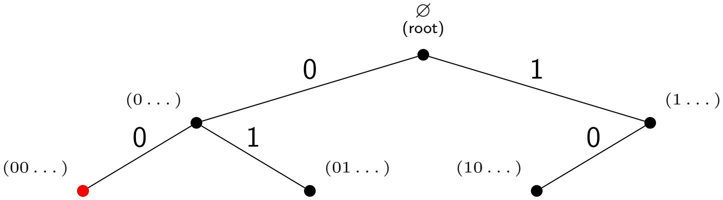

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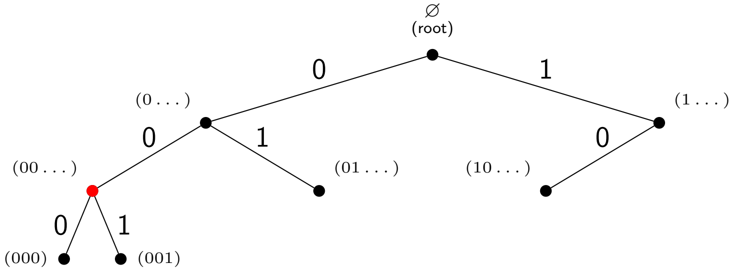

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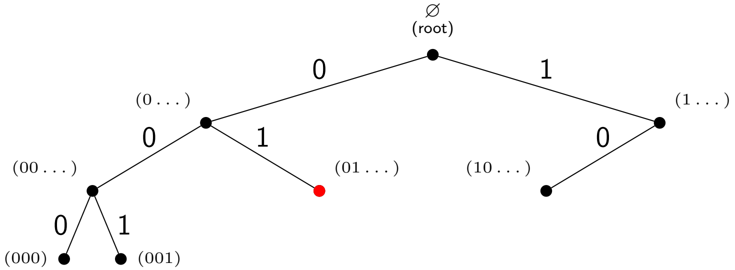

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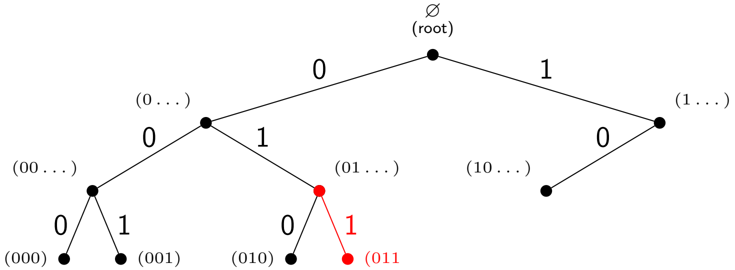

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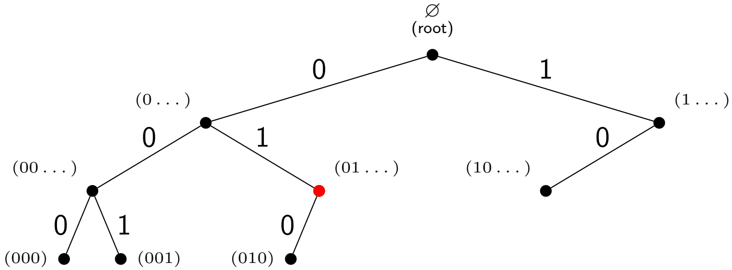

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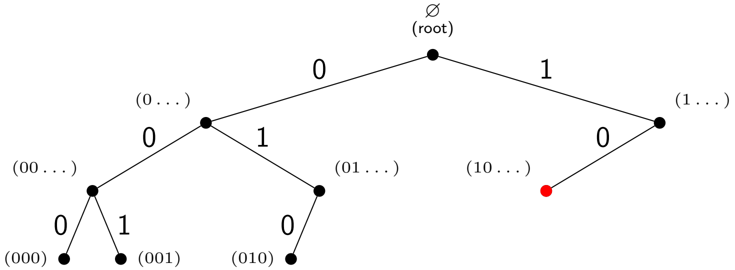

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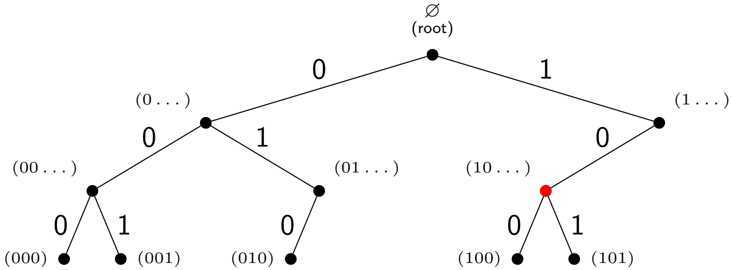

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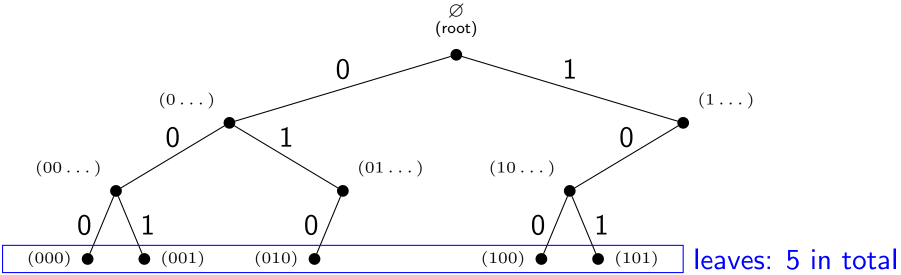

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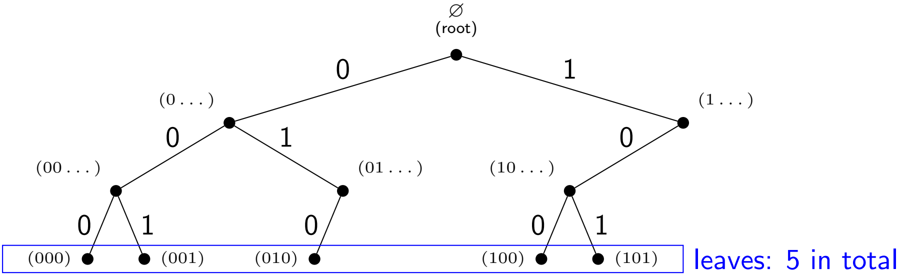

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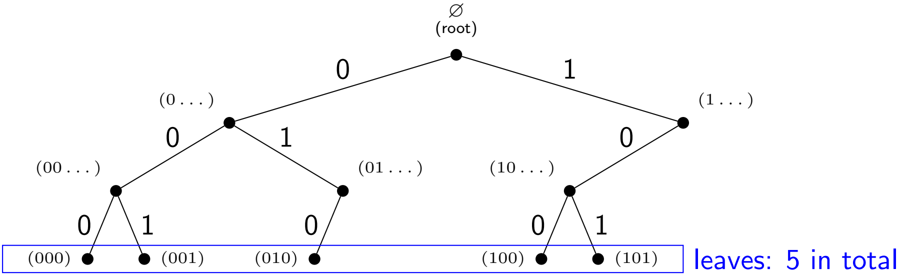

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You try: Exercise 29

Applications of recurrence relations

Recall that a recursive definition for a sequence is an expression of a_n using the previous terms:

For example: $\underbrace{a_n = 3a_{n-1} + a_{n-3} + 1}_{\text{recurrence relation}}$ $\underbrace{a_0 = 1,\ a_1 = 15,\ a_2 = 0}_{\text{initial conditions}}$

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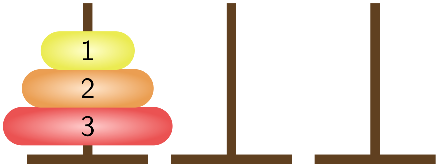

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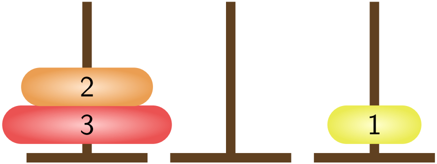

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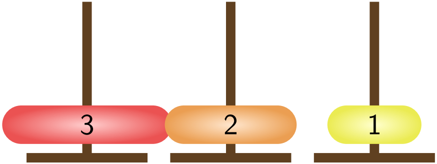

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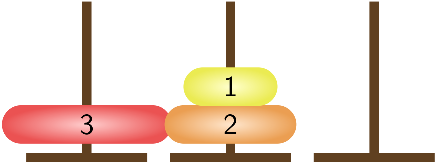

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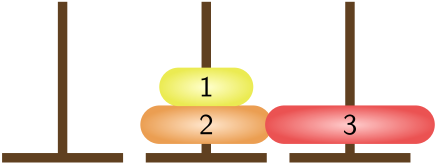

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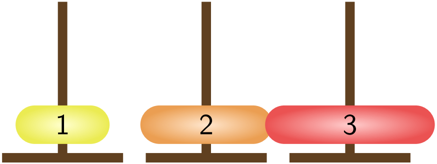

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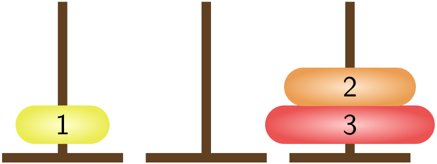

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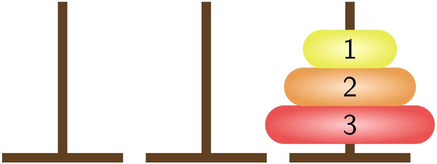

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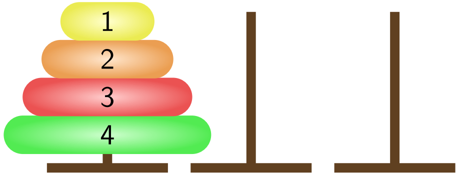

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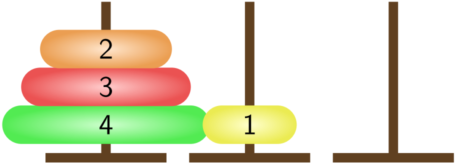

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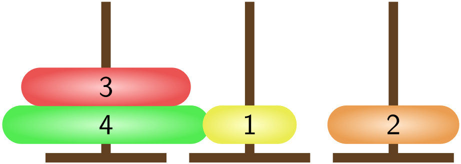

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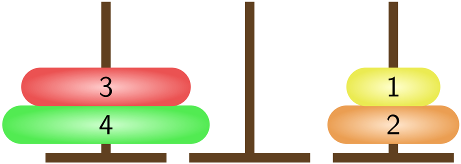

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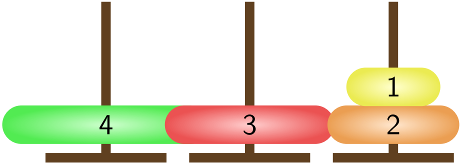

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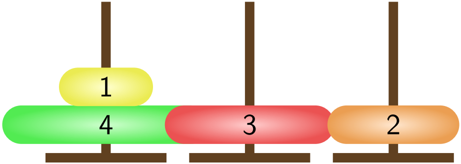

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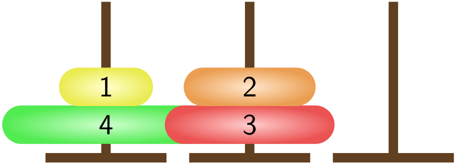

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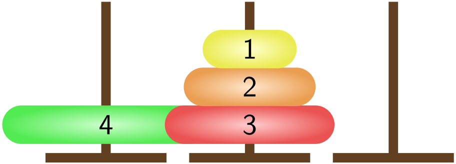

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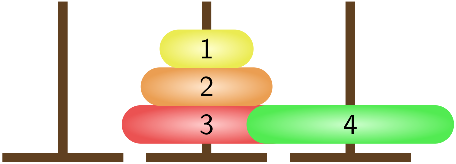

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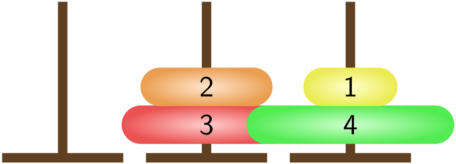
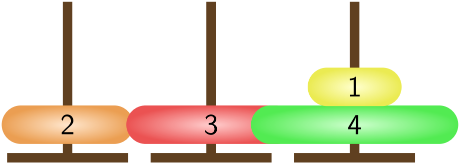


Moved disc from pole 2 to pole 3.

Towers of Hanoi: Start with n discs of different sizes on the left-most of three pegs, in increasing order of size top-to-bottom. Move discs from pole to pole one at a time. End with all n discs on the right-most of three pegs, again in decreasing order top-to-bottom.



Moved disc from pole 2 to pole 1.

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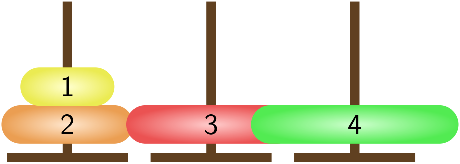
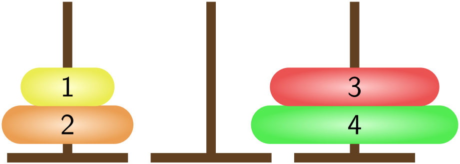


Moved disc from pole 3 to pole 1.

Towers of Hanoi: Start with n discs of different sizes on the left-most of three pegs, in increasing order of size top-to-bottom. Move discs from pole to pole one at a time. End with all n discs on the right-most of three pegs, again in decreasing order top-to-bottom.



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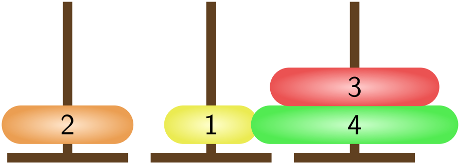

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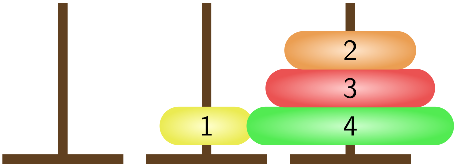

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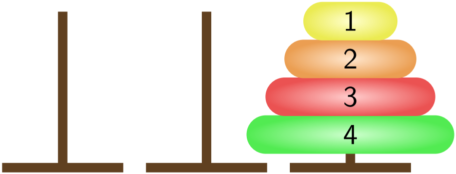

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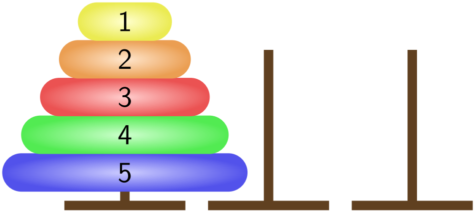

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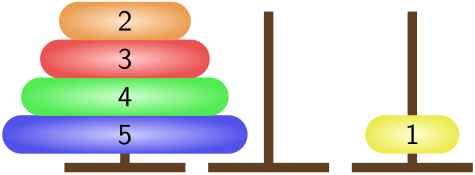

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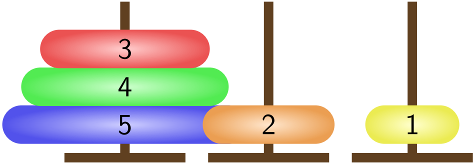

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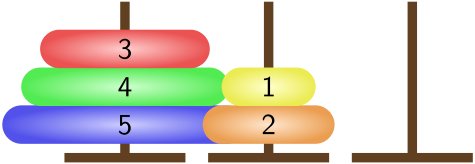

Moved disc from pole 3 to pole 2.

Solution: We solved these by first piling the all but one of the discs onto pole 2 (which takes the same number of moves are piling all but one onto pole 3); then we moved the biggest disc; then we moved the $n-1$ discs on top.

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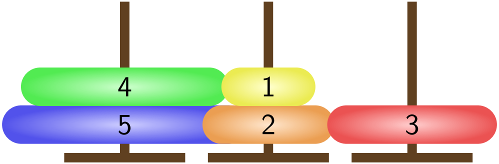

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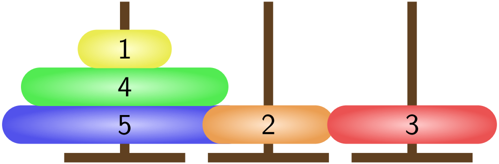

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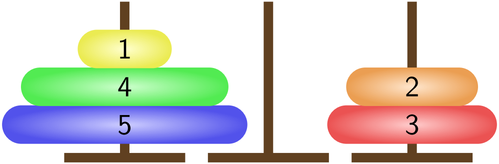

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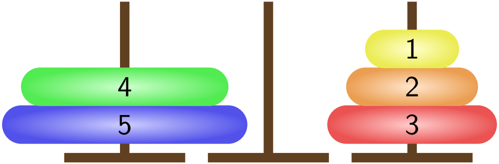

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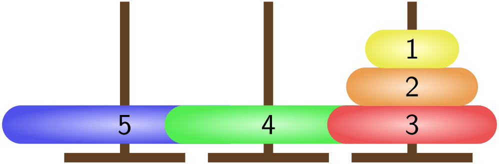

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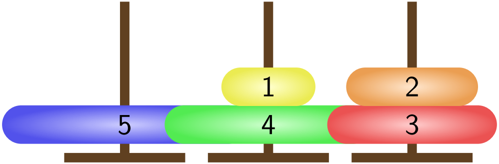

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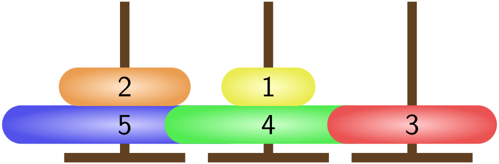

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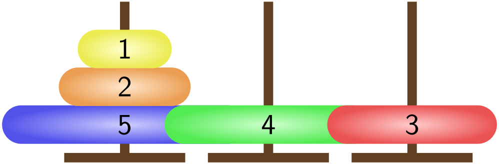

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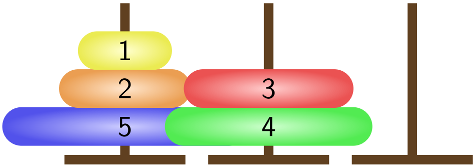

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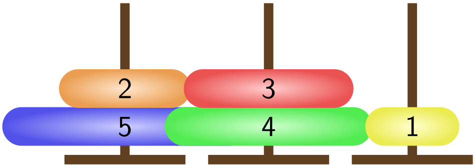

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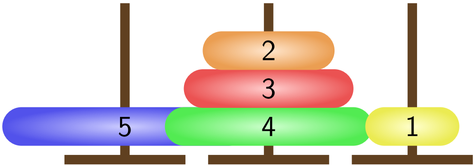

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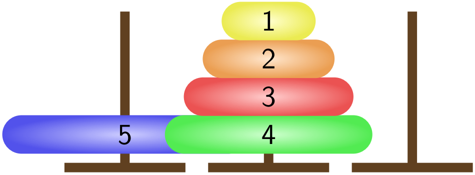

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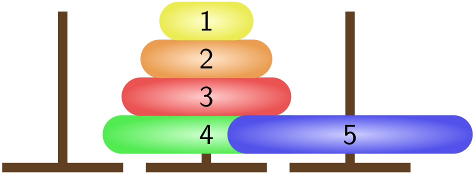

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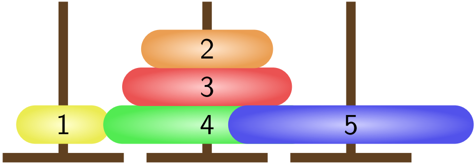

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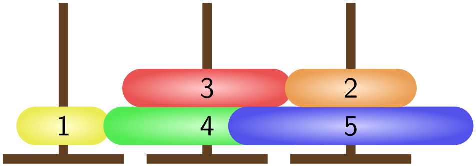

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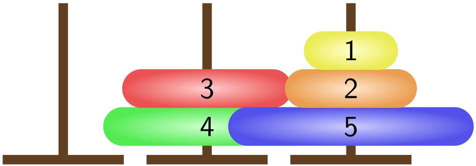

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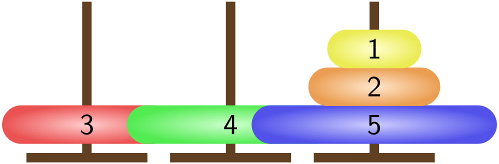

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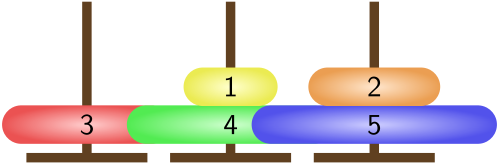

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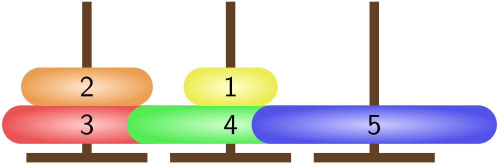

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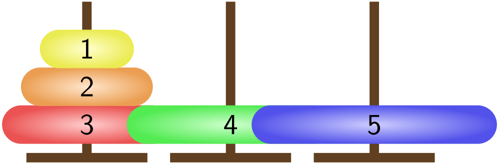

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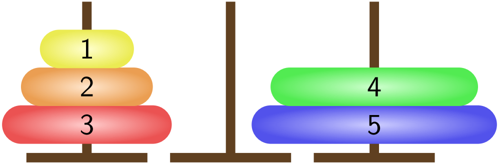

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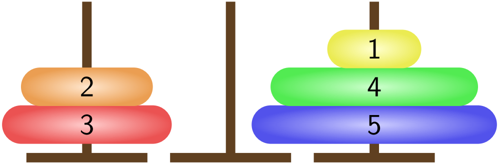

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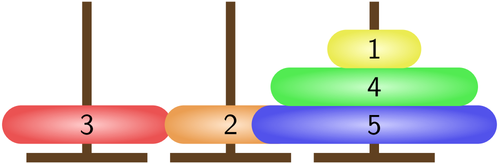
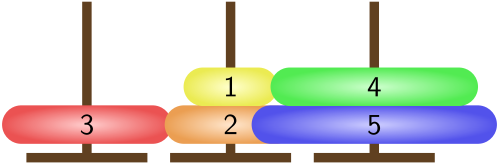


Moved disc from pole 1 to pole 2.

Solution: We solved these by first piling the all but one of the discs onto pole 2 (which takes the same number of moves are piling all but one onto pole 3); then we moved the biggest disc; then we moved the $n-1$ discs on top.

Towers of Hanoi: Start with n discs of different sizes on the left-most of three pegs, in increasing order of size top-to-bottom. Move discs from pole to pole one at a time. End with all n discs on the right-most of three pegs, again in decreasing order top-to-bottom.



Moved disc from pole 3 to pole 2.

Solution: We solved these by first piling the all but one of the discs onto pole 2 (which takes the same number of moves are piling all but one onto pole 3); then we moved the biggest disc; then we moved the $n-1$ discs on top.

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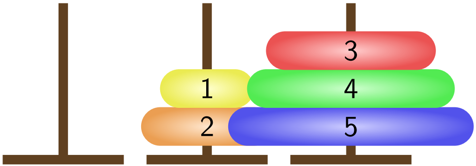

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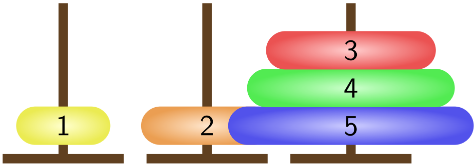

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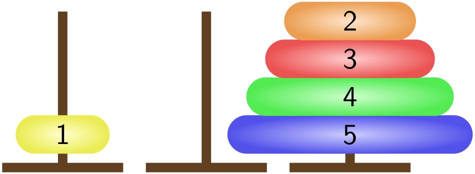

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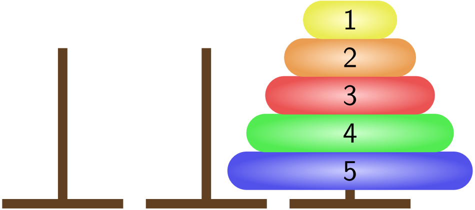

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You try: Exercise 30