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[Hint: "...they have the same size" is not the right answer.]

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X and Y are in bijection with eachother.

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there is a bijective function $f: X \to Y$.

“Outline a proof by induction that $\sum_{i=1}^{n} = n(n+1)/2$.

Inductive step

$$\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$$

$$1+2+\cdots+n+n+1 = \frac{(n+1)(n+2)}{2}$$

$$-(n+1) \qquad\qquad -(n+1)$$

$\times 2$:

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$$2\sum_{i=1}^{n} i = n^2+3n+2-n-2$$

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Problem: Manipulating formulas from "what you want" to "something true" isn't very reliable.

Bad example.

Claim: $-1 = 1$

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If $-1 = 1$, then

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If $-1 = 1$, then

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What went wrong:

We proved that

$$\text{“}-1 = 1 \text{ implies } 1 = 1\text{”},$$

which *is* (strangely enough) true. A false statement can imply a true statement.

We **did not** show that $-1 = 1$.

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Good:

- Inductive step: Fix $n \geq 1$, and assume $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ (for that n).

Then

$$\sum_{i=1}^{n+1} i = \underbrace{1 + 2 + \cdots + n}_{\sum_{i=1}^{n} i} + (n+1)$$

$$= \frac{n(n+1)}{2} + (n+1) \quad \text{(by the induction hypothesis)}$$

$$= \frac{n^2 + n + 2n + 2}{2} = \frac{(n+1)(n+2)}{2}.$$

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One more problem:

$P(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ versus $P(n): \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

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Problem: $P(n)$ is a *statement*, like “avocados are green”, and can’t equal anything.

On the other hand, $n(n+1)/2$ is an *expression*, which is basically a noun, like “avocados”.

Last time: Counting rules.

Product: If a procedure can be broken into a sequence of two tasks, and there are n_1 ways to do the first task, and for each of these ways of doing the first task, there are n_2 ways to doing the second task, then there are $n_1 n_2$ total distinct outcomes.

Sum: If a procedure can be done either in one of n_1 ways or in one of n_2 ways, where there is no overlap in the n_1 and n_2 ways, then there are $n_1 + n_2$ total distinct outcomes.

Subtraction/Inclusion-exclusion: If a procedure can be done either in one of n_1 ways or in one of n_2 ways, but there are n_3 overlapping outcomes, then there are $n_1 + n_2 - n_3$ total distinct outcomes.

Division: If a procedure can be done in n ways, but that procedure produces each outcome in d different ways, then there are actually n/d distinct outcomes.

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Remember: Every rule depends on making up a "procedure" for counting, and then applying the rules according to that procedure!!
(Take it from this expert: *Never* just plug stuff into a formula – make up a story for counting things one step at a time, and *then* try to count.)

Section 6.3: Permutations and combinations

The choose function is

$$C(n,k) = \binom{n}{k} = \#\{ \text{ ways to choose } k \text{ objects from } n \text{ } \},$$

read "n choose k".

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Let's start with an example...

$$C(n,k) = \binom{n}{k} = \#\{\text{ ways to choose } k \text{ objects from } n\ \}$$

Let's start with an example: $n = 7$, $k = 3$.

Step 1: How many ways are there to select 3 things from 7 in order (no replacement)?

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Ans: $7 * 6 * 5$.

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Rewriting, notice that

$$7! = \big(7 * 6 * 5\big) * \underbrace{\big(4 * 3 * 2 * 1\big)}_{(7-3)!=4!}$$

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A k-permutation of n objects is a choice of k things from n **in order**.

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We call counting problems that call for <u>unordered</u> selection "combination problems".
We call counting problems that call for <u>ordered</u> selection "permutation problems".

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Some examples:

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Some examples:

(1) How many ways can one pick a president, vice president, and secretary in a club of 20 people?

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You try: Exercise 22

Pigeonhole principle - §6.2

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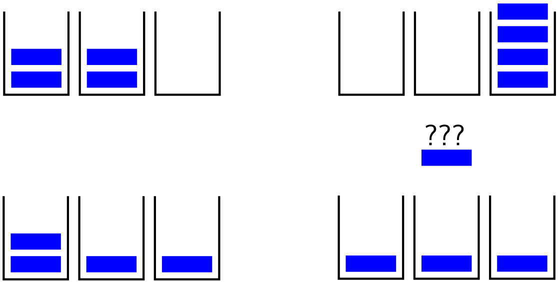

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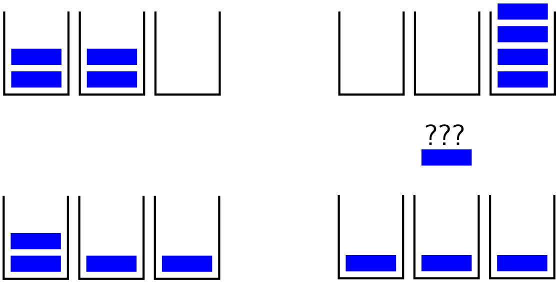

Proof (by contradiction).

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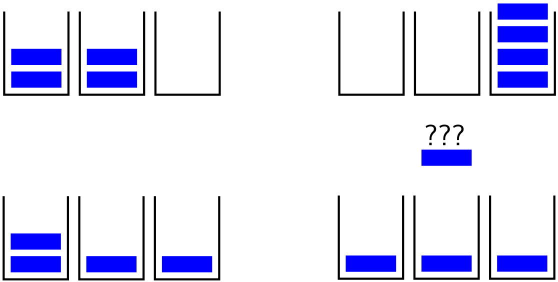

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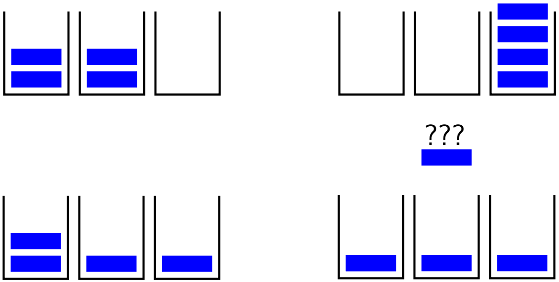

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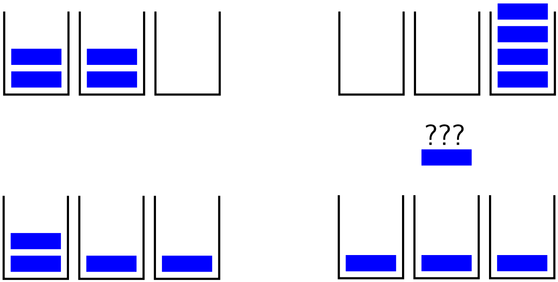

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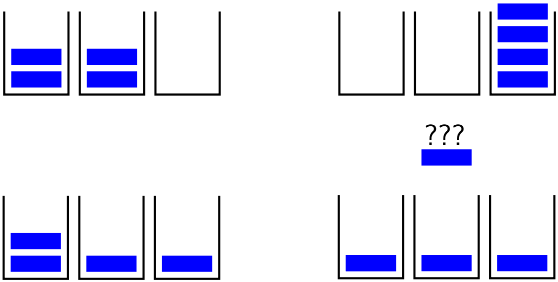

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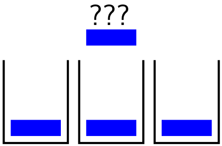

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If A is a set of size $k+1$ and B is a set of size k, then there is no injective function $f : A \to B$.

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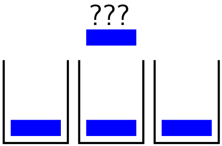

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Pigeonhole Principle: "if $k + 1$ objects are placed into k boxes, then at least one box contains two or more objects."

Ex: In any group of 367 people, at least two of those people have the same birthday.

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Ex: In any group of 367 people, at least two of those people have the same birthday.

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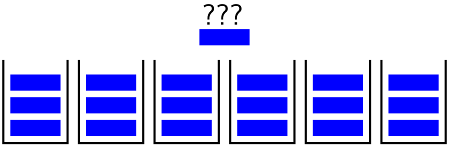

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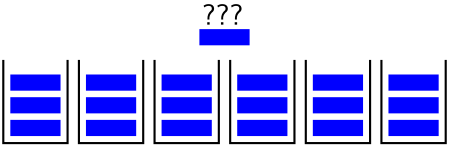

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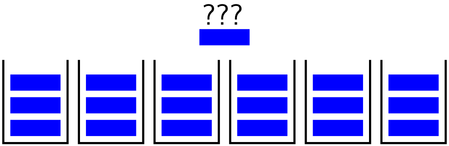

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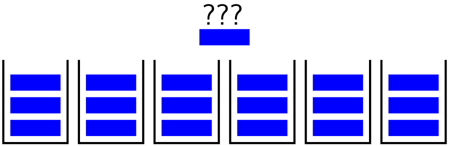

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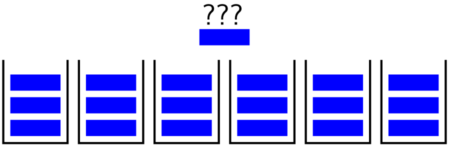

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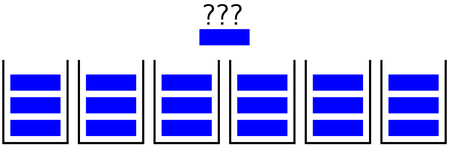

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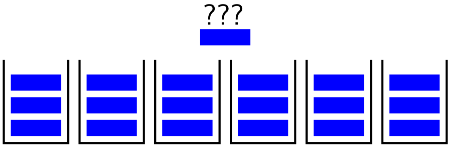

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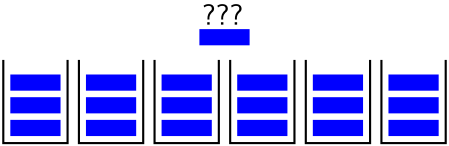

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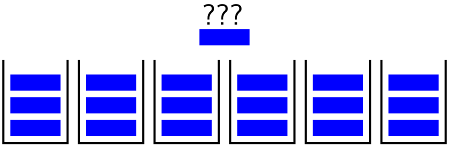

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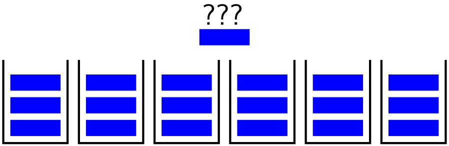

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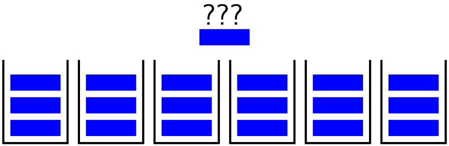

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Example: What is the minimum number of students required in a class to ensure that at least ten people will receive the same grade (if the grade options are just A,B,C,D,F)?

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($\lceil N/5 \rceil \geqslant 10$)
But

$$\lceil N/5 \rceil \geqslant 10 \quad \text{exactly when} \quad N/5 > 9.$$

So the question comes down to finding the least integer N such that $N > 5 * 9$: $\boxed{N = 5 * 9 + 1}$

The Generalized Pigeonhole Principle says that if N objects are placed into k boxes, then at least one box contains $\lceil N/k \rceil$ objects.

Example: How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit appear?

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Answer:

Here, the suits are the “boxes”, of which there are 4. ($k = 4$)

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Answer:

Here, the suits are the "boxes", of which there are 4. $(k = 4)$
Need at least one of the boxes to have at least three cards.

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Answer:

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Need at least one of the boxes to have at least three cards.
$(\lceil N/4 \rceil \geqslant 3)$

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So the question comes down to finding the least integer N such that $N > 4 * 2$: $\boxed{N = 4 * 2 + 1}$

The Generalized Pigeonhole Principle says that if N objects are placed into k boxes, then at least one box contains $\lceil N/k \rceil$ objects.

NON-Example: How many cards must be selected from a standard deck of 52 cards to guarantee that at least three hearts appear?

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NON-Example: How many cards must be selected from a standard deck of 52 cards to guarantee that at least three hearts appear?

Answer: Since there are 39 cards that are not hearts, we would need to pull at least $39 + 3 = 42$ cards to ensure at least three hearts.

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So if we have 25 million phone numbers, at least

$$\lceil 25\text{mil}/8\text{mil} \rceil = 4$$

will have the same 7-digit phone number.

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will have the same 7-digit phone number. Therefore we need $\boxed{4}$ area codes.

You try: Exercise 23