Welcome to Math 365!

Warm up:

1. How many ways can you choose 2 things from a set of 4? (Example, pick a committee of 2 people from a group of 4. This is different from the number of ways to choose a president and a vice president from a group of 4 people.)
2. How many ways can you choose 3 things from a set of 5?
3. Explain why there are exactly the same number of ways to choose 1 thing from a set of 5 as there are ways to choose 4 things from a set of 5.
4. How many ways are there to choose 3 things from a set of 3? 4 things from a set of 4? 5 things from a set of 5?
5. How many ways are there to choose 0 things from a set of 3? 0 things from a set of 4? 0 things from a set of 5?
6. Expand:

$$(1+x)^2 = 1 + 2x + x^2$$
$$(1+x)^3 =$$
$$(1+x)^4 =$$

Pascal's Triangle

Start and end each row with a 1. The ith row (starting with the 0th row) has $i + 1$ entries. The middle entries are acquired by adding successive entries in the previous row.

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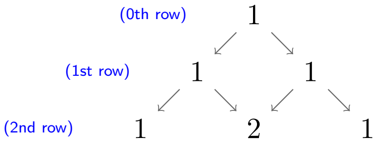

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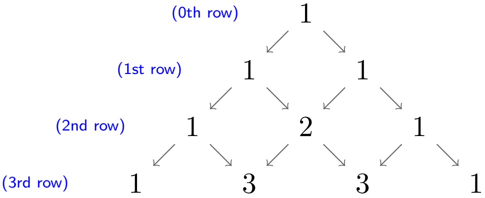

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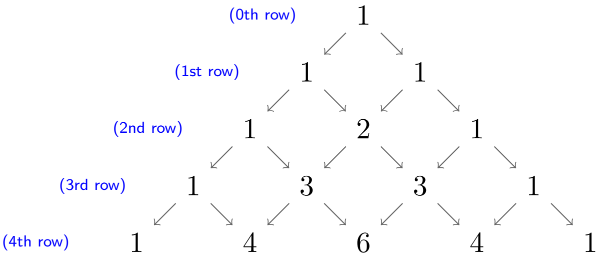

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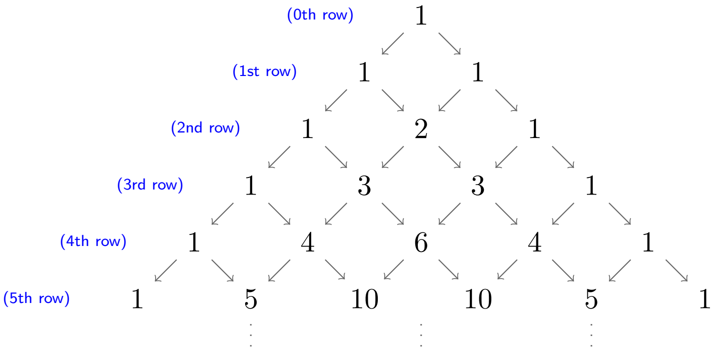

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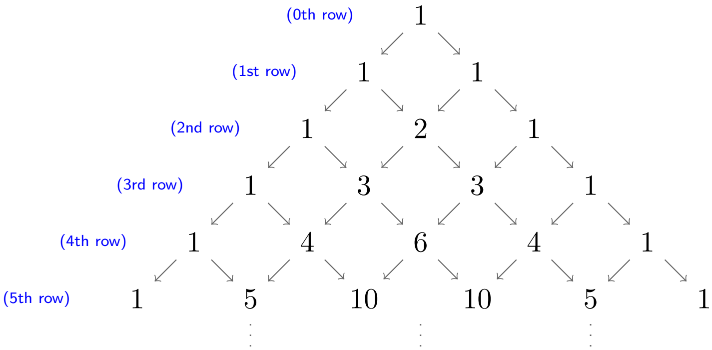

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ways to choose k things from n = "n choose k" $= \binom{n}{k}$.

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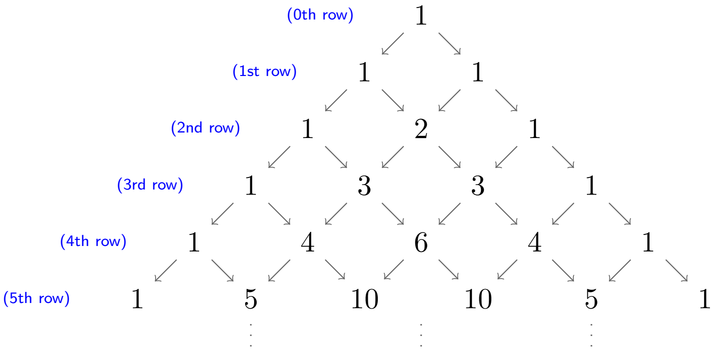

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Claim: $\binom{n}{k}$ is the kth entry of the nth row of Pascal's triangle

Course info

Me: Professor Daugherty, zdaugherty@gmail.com
Website:
https://zdaugherty.ccnysites.cuny.edu/teaching/m365s19/
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Homework: due on Wednesdays in class, graded by completion. Posted on course website. FINAL DRAFTS.
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Homework 0: Before class on Monday 2/4, send me an email at zdaugherty@gmail.com with subject line "Math 365: Homework 0", answering the questions outlined on the website.

Course expectations

- Read posted sections before class, and **bring your own copy of daily handouts and notes** (posted night before class).
- Come to class, participate, ask questions, work (possibly together) on in-class exercises.
- Come to office hours at least once in the semester (worth one homework assignment). If you can't make my office hour, make an appointment.
- Out of class studying and work should be 2–3 times the amount of time spent in class ($6.5 <$ hours/week). Find classmates to study and work with!
- Hand in "final draft" homework, on time. Get good practice with writing; using words and complete sentences—see Writing Exercise. Ok to work with other people, but write-ups should be your own. Homework submitted in LaTeXreceives 10% extra credit.
- If there are accessibility accommodations or exam conflicts to be organized, contact me as soon as possible.
- If you send me email, use complete sentences and be specific (ok to send pics of work!).

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Some special sets:

notation	definition	some terms
$\mathbb{Z}$	Integers	$0, \pm 1, \pm 2$
$\mathbb{Z}^+, \mathbb{Z}_{>0}$	Positive integers	$1, 2, 3$
$\mathbb{Z}_{\geqslant 0}$ $(\mathbb{N})$	Natural numbers	$0, 1, 2, 3$
$\mathbb{Q}$	Rational numbers (fractions)	$0, 1, -1/2, 15/1004$
$\mathbb{R}$	Real numbers	$0, 1, 1/3, \pi, -\sqrt{2}$
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Notice: $\mathbb{C} \supsetneq \mathbb{R} \supsetneq \mathbb{Q} \supsetneq \mathbb{Z} \supsetneq \mathbb{Z}_{\geqslant 0} \supsetneq \varnothing$.

(Notation: $\subseteq$ means subset, $\subsetneq$ means proper subset, and $\nsubseteq$ means not a subset. The symbol $\subset$ is unclear, and we try not to use it in this class.)

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$$\Big\{ \underbrace{\qquad\qquad}_{\text{objects}} \Big| \underbrace{\qquad\qquad}_{\text{conditions}} \Big\}.$$

Read $|$ as "such that" or "that satisfy". Also useful:

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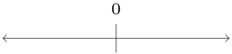

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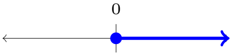

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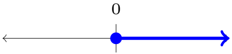

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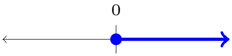

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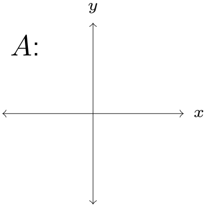

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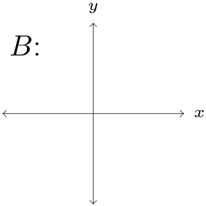

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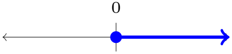

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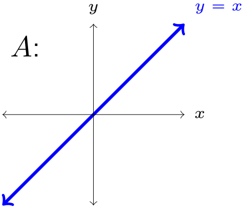

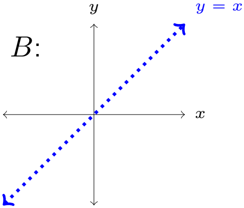

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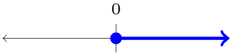

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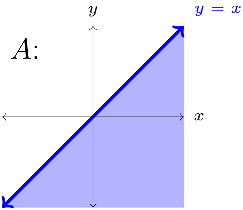

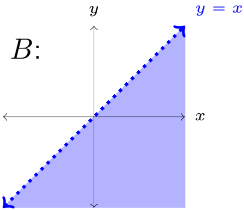

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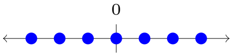

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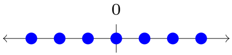

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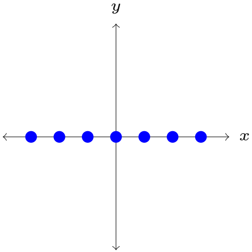

Let A and B be sets.

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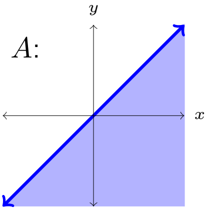
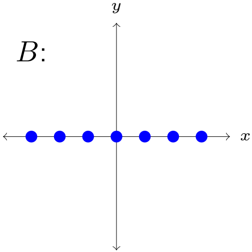
(vii) If A has a finite number of distinct elements, the cardinality of A, denoted $|A|$, is the number of those elements. Otherwise, we say A is infinite.

Example

Let $A = \mathbb{R}^2_{x \geqslant y}$, $B = \mathbb{Z} \times \{0\}$, and $U = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.





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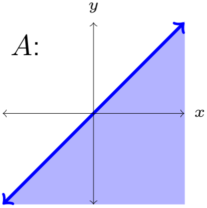
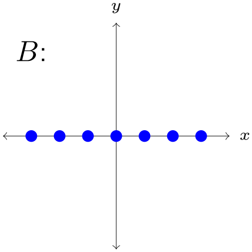
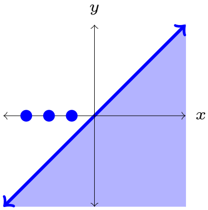




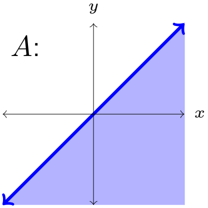
Then we have

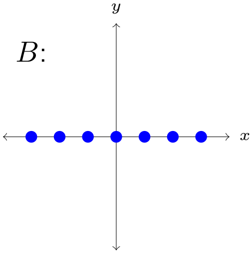
$$A \cup B = \{x \in U \mid x \in A \text{ or } B\} =$$



Example

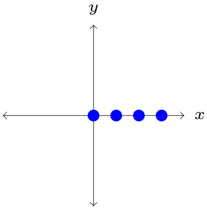
Let $A = \mathbb{R}^2_{x \geqslant y}$, $B = \mathbb{Z} \times \{0\}$, and $U = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.





Then we have

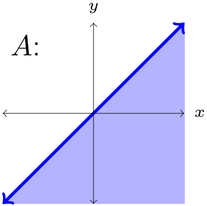
$$A \cap B = \{x \in U \mid x \text{ in both } A \text{ and } B\} =$$



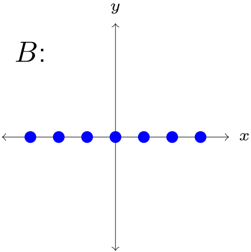
Example

Let $A = \mathbb{R}^2_{x \geqslant y}$, $B = \mathbb{Z} \times \{0\}$, and $U = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.

A:

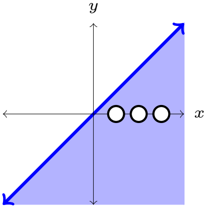


B:



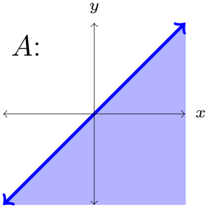
Then we have

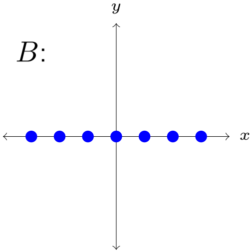
$$A - B = \{x \in A \mid x \notin B\} =$$



Example

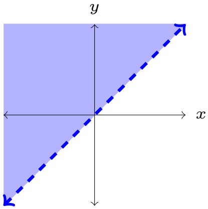
Let $A = \mathbb{R}^2_{x \geqslant y}$, $B = \mathbb{Z} \times \{0\}$, and $U = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.





Then we have

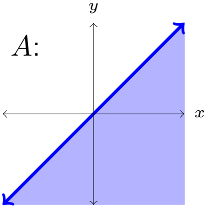
$$\overline{A} = U - A =$$



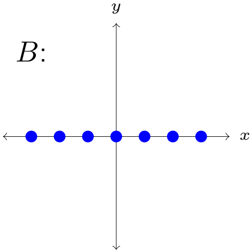
Example

Let $A = \mathbb{R}^2_{x \geqslant y}$, $B = \mathbb{Z} \times \{0\}$, and $U = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$.

A:



B:



Then we have

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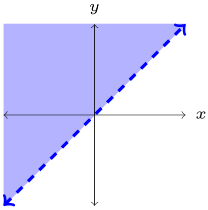

You try: Exercise 1