Math 345 – Wednesday 9/27/17

Exercise 21. Use Fermat's Little Theorem to do the following without the use of a computer (show your work!).

(a) Find the least residue of 9^{794} (mod 73).

(b) Solve $x^{86} \equiv 6$ (mod 29).

(c) Solve $x^{39} \equiv 3$ (mod 13).

Exercise 22. Recall the quantity $(p-1)!$ (mod p) appeared in our proof of Fermats Little Theorem (without actually having to compute it).

(a) Use a computer to calculate $(p-1)!$ (mod p) for primes p up to 13.

(b) Make a conjecture for what $(p-1)!$ (mod p) is in general, and prove it.
   [Hint: Do a few examples by hand – say for $p = 2, 3$, and 5, and try to discover why $(p-1)!$ (mod p) has the value it does. Then generalize your observation to prove the formula for all values of p.]

(c) Compute the value of $(m-1)!$ (mod m) for some small values of m that are not prime ($m = 4, 6, \dots$). Do you find the same pattern as you found for primes? Do you see any pattern?