Exercise 11.

(a) Describe all integer solutions to each of the following equations.

$$105x + 121y = 1 \quad \text{and} \quad 12345x + 67890y = \gcd(12345, 67890)$$

(first find one solution, and go from there).

(b) Show that, for $a, b \in \mathbb{Z}_{\neq 0}$, and any $x, y \in \mathbb{Z}$, that

$$\text{if } d|a \text{ and } d|b \quad \text{then} \quad d|(ax + by).$$

(Do *not* assume that $ax + by = \gcd(a, b)$. There are lots of other integral combinations of a and b.)

(c) Suppose that $\gcd(a, b) = 1$. Prove that for every integer c, the equation $ax + by = c$ has a solution in integers x and y

(d) Now, in general, if $\gcd(a, b) = g$, what integers c come in the form $c = ax + by$?
(See the spreadsheet from lecture–try plugging in different values for a and b and observing which values appear in the table. Then answer in general, and prove your claim.)

Exercise 12.

(a) Find integers x, y, and z that satisfy the equation

$$6x + 15y + 20z = 1.$$

(b) Under what conditions on a, b, c is it true that the equation

$$ax + by + cz = 1$$

has an integer solution? (So that $x, y, z \in \mathbb{Z}$.)
Describe a general method of finding a solution when one exists.

(c) Use your method from (b) to find a solution in integers to the equation

$$155x + 341y + 385z = 1.$$

Attach at the end of Homework 3:
At the end of your write-up, include the following, labeling this as **"Writing exercise"**.

(a) Mark up this written homework assignment, showing where you followed or failed to follow the mechanical and stylistic issues outlined in *Communicating Mathematics through Homework and Exams*. How did you improve this week over homeworks 1 and 2? How might you improve in the future?

(b) List three or more ways that you succeeded or failed at following the advice in *Some Guidelines for Good Mathematical Writing*. How did you improve this week over homeworks 1 and 2? How might you improve in the future?

To receive credit for this assignment, you must complete this exercise.