Math 345 – Monday 9/11/17

Exercise 8. Set up a computer program or spreadsheet to compute the $\gcd(a, b)$ for any positive integers a and b. To check that your answer is correct, plug in the values $a = 100$ and $b = 36$, and compare your q and r values to those from lecture.

Now, compute the prime factorizations of the following a and b values (ok to use a calculator), and use those to compute $\gcd(a, b)$. Then plug into your program/spreadsheet to verify your answer. Report how many steps the Euclidean algorithm took in each example (i.e. what is n?)

(a) $a = 242$, $b = 25$;
(b) $a = 5390$, $b = 504$.

Exercise 9. Recall from lecture that executing the Euclidean algorithm for $a = 100$ and $b = 36$ gives the following equations:

$$100 = 36 * 2 + 28, \tag{E1}$$
$$36 = 28 * 1 + 8, \tag{E2}$$
$$28 = 8 * 3 + 4, \tag{E3}$$
$$8 = 4 * 2 + 0. \tag{E4}$$

(a) Follow these steps to express 4 as an *integer combination* of 100 and 36, i.e., find (possibly negative) integers x and y such that $100x + 36y = 4$:
   (i) Use equation (E3) to express 4 as an integer combination of 8 and 28 (find integers x and y such that $8x + 28y = 4$).
   (ii) Use equation (E2) to express 8 as an integer combination of 28 and 36 (find integers x and y such that $28x + 36y = 8$).
   (iii) Use equation (E1) to express 28 as an integer combination of 36 and 100 (find integers x and y such that $36x + 100y = 28$).
   (iv) Plug your equation from part (ii) into your equation in part (i), expanding and simplifying, to express 4 as an integer combination of 28 and 36 (find integers x and y such that $36x + 28y = 4$).
   (v) Plug your equation from part (iii) into your equation in part (iv), expanding and simplifying, to express 4 as an integer combination of 36 and 100 (find integers x and y such that $100x + 36y = 4$).

(b) Use your computer calculations from Exercise 8(b) to write out the equations for the Euclidean algorithm (like those in (E1)–(E4)). Then use those to write $\gcd(242, 25)$ as an integer combination of 242 and 25, using the same strategy as in part (a).

(c) Make an argument justifying the following claim:
For any positive integers a and b, there exist integers x and y satisfying $\gcd(a, b) = ax + by$.

Exercise 10. A number ℓ is called a *common multiple* of positive integers a and b if $a|\ell$ and $b|\ell$. The smallest (positive) such ℓ is called the *least common multiple* of a and b, denoted $\text{lcm}(a, b)$. For example, $\text{lcm}(3, 7) = 21$ and $\text{lcm}(12, 66) = 132$.

(a) Complete the following table of values (using your program/spreadsheet to compute the gcd).

a	b	ab	$\gcd(a, b)$	$\text{lcm}(a, b)$
12	8			
30	20			
68	51			
23	18			

Try to surmise a relationship between a, b, $\gcd(a, b)$, and $\text{lcm}(a, b)$.

(b) Give an argument proving that the relationship you found at the end of part (a) is correct for all a and b.

(c) Use your result in (b), along with your gcd calculator to $\text{lcm}(301337, 307829)$.

(d) Suppose that $\gcd(a, b) = 18$ and $\text{lcm}(a, b) = 720$. What are the possibilities for the values of a and b?

Attach at the end of Homework 3:

At the end of your write-up, include the following, labeling this as **"Writing exercise"**.

(a) Mark up this written homework assignment, showing where you followed or failed to follow the mechanical and stylistic issues outlined in *Communicating Mathematics through Homework and Exams*. How did you improve this week over homework 1? How might you improve in the future?

(b) List three or more ways that you succeeded or failed at following the advice in *Some Guidelines for Good Mathematical Writing*. How did you improve this week over homework 1? How might you improve in the future?

To receive credit for this assignment, you must complete this exercise.