Exercise 6. Recall, we get every Pythagorean triple (a, b, c) with b even from the formula
$$(a, b, c) = (u^2 - v^2, 2uv, u^2 + v^2)$$
by substituting in different integers for u and v. For example, $(u, v) = (2, 1)$ gives the smallest triple $(3, 4, 5)$.

(a) If u and v have a common factor, explain why (a, b, c) will not be a primitive Pythagorean triple.

(b) Find an example of integers $u > v > 0$ that do not have a common factor, yet the Pythagorean triple $(u^2 - v^2, 2uv, u^2 + v^2)$ is not primitive.

(c) Make a table of the Pythagorean triples that arise when you substitute in all values of u and v with $1 \leq v < u \leq 10$.

(d) Using your table from (c), find some simple conditions on u and v that ensure that the Pythagorean triple $(u^2 - v^2, 2uv, u^2 + v^2)$ is primitive.

(e) Prove that your conditions in (d) really work.

Exercise 7. Rational points on other curves.

(a) Use the lines through the point $(1, 1)$ to describe all the points on the circle $x^2 + y^2 = 2$ whose coordinates are rational numbers. Be sure to draw pictures.

(b) Provide 2 illustrative examples of the results you acquired in part (a).

(c) What goes wrong if you try to apply the same procedure to find all the points on the circle $x^2 + y^2 = 3$ with rational coordinates?

Attach at the end of Homework 2:

At the end of your write-up, include the following, labeling this as **"Writing exercise"**.

(a) Mark up this written homework assignment, showing where you followed or failed to follow the mechanical and stylistic issues outlined in *Communicating Mathematics through Homework and Exams*. How did you improve this week over homework 1? How might you improve in the future?

(b) List three or more ways that you succeeded or failed at following the advice in *Some Guidelines for Good Mathematical Writing*. How did you improve this week over homework 1? How might you improve in the future?

To receive credit for this assignment, you must complete this exercise.