

Math 345 – Wednesday 11/08/17

Exercise 39. Use the Law of Quadratic Reciprocity to decide whether a is a square mod b .

- (a) $a = 85, b = 101$
- (b) $a = 29, b = 541$
- (c) $a = 101, b = 1987$
- (d) $a = 31706, b = 43789$

Exercise 40. Does the congruence

$$x^2 - 3x - 1 \equiv 0 \pmod{31957}$$

have any solutions?

[Hint. Use the quadratic formula to find out what number you need to take the square root of modulo the prime 31957.]

Exercise 41. Let p be a prime satisfying $p \equiv -1 \pmod{4}$ and suppose that a is a quadratic residue modulo p .

- (a) Show that $x = a^{(p+1)/4}$ is a solution to the congruence $x^2 \equiv a \pmod{p}$.
(This gives an explicit way to find square roots modulo p for primes congruent to $-1 \pmod{4}$.)
- (b) Find a solution to the congruence $x^2 \equiv 7 \pmod{787}$.
(Your answer should lie between 1 and 786.)