Math 345 – Wednesday 10/11/17

Exercise 27.

(a) Prove that there are infinitely many primes that are congruent to 5 (mod 6). [Use the proof that there are infinitely many primes that are congruent to 3 (mod 4) as your guide.]

(b) Try to use the same idea (considering $N = 5p_1p_2 \cdots p_\ell + 4$) to show that there are infinitely many primes congruent to 4 (mod 5). What goes wrong? In particular, what happens if you start with $\{19\}$ and try to make a longer list?

Exercise 28.

(a) In class, you were given two finite arithmetic progressions consisting of prime numbers. Moreover, they were maximal (neither the previous number nor the next number in the progressions are prime). Find three more maximal arithmetic progressions of prime numbers—all of length 3 or more, with at least one of length 4 or more. [Don't just look up the answer!]

(b) Read the wikipedia article on the "Green-Tao theorem", and summarize it. What does the theorem mean? What are its implications? What are its limitations? In particular, why is the section on "Numerical work" important?

(c) Read the introduction section of the wikipedia article on "Van der Waerden's theorem" and summarize it (no ned to summarize the sections on its proofs).