Exercise 25.

(a) Compute $\phi(97)$ and $\phi(8800)$.

(b) For $n \geq 3$, show $\phi(n)$ is even.

(c) Fill in the blank and prove: $\phi(n)$ *is a multiple of 4 if and only if* _____.

(d) Suppose that p_1, p_2, ..., p_r are the distinct primes that divide n (for example, if $n = 7000$, then this list is $2, 5$, and 7). Use what we already know about $\phi(n)$ to prove that

$$\phi(n) = n \left(1 - \frac{1}{p_1} \right) \left(1 - \frac{1}{p_2} \right) \cdots \left(1 - \frac{1}{p_r} \right).$$

Use this formula to double check the value of $\phi(7000)$ (calculated in class), and to compute 1000000. Compare your answer to the other formula for $\phi(n)$.

(e) Find at least one solution to $x^{8644} = 16 \pmod{2025}$.

Exercise 26.

(a) Find an x that satisfies both $x \equiv 3 \pmod 7$ and $x \equiv 5 \pmod 9$.

(b) Find an x that satisfies both $x \equiv 3 \pmod{37}$ and $x \equiv 1 \pmod{87}$.