Computing data with spreadsheets

Example: Computing triangular numbers and their square roots.

Recall, we showed $1 + 2 + \cdots + n = n(n+1)/2$.

Enter the following into the corresponding cells:

A1: n	B1: triangle	C1: sqrt
A2: 1	B2: =A2*(A2+1)/2	C2: =SQRT(B2)
A2: =A2+1	B2: =A3*(A3+1)/2	C3: =SQRT(B3)

The result should look like

	A	B	C
1	n	trangle	sqrt
2	1	1	1
3	2	3	1.73205081
4			

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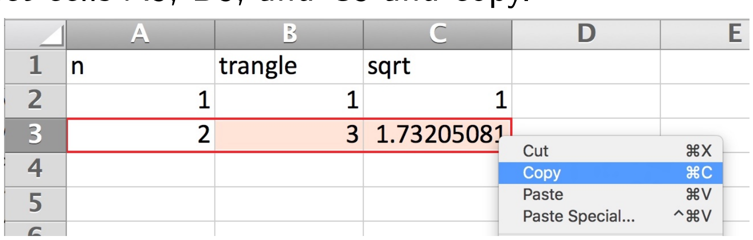
Next, select cells A3, B3, and C3 and copy.



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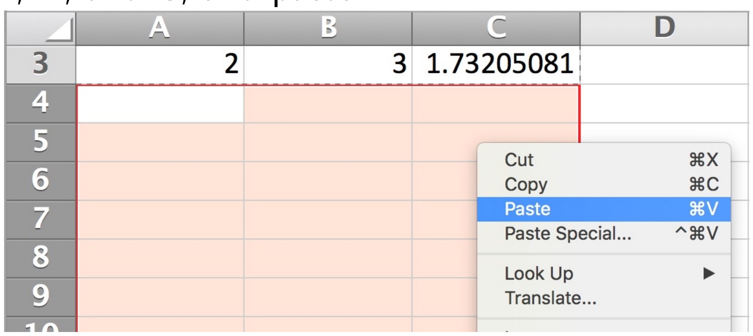

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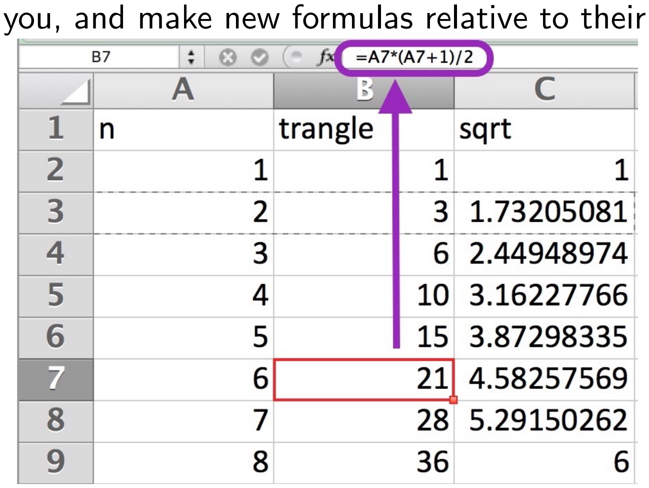
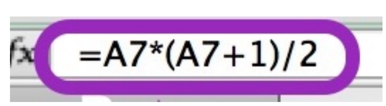


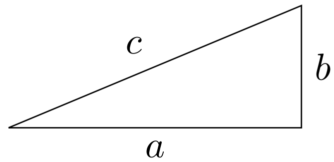


If you *don't* want it to adjust part of a formula with shifts, add a $ in front of the row number, cell number, or both: A$2, $A2, or A2.

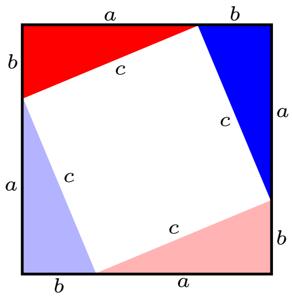
Pythagorean Triples

The Pythagorean theorem says the lengths of the sides of a right triangle satisfy the following:

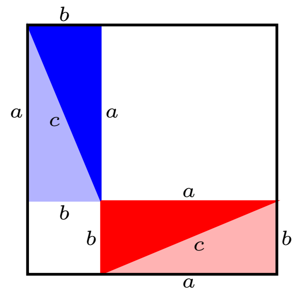
$$a^2 + b^2 = c^2$$



Geometric proof: Compare the area of the white spaces in



versus



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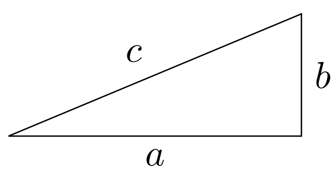

Number theorists ask: Are there integer solutions? If so, are there infinitely many integer solutions? Do they all follow some common pattern, or are the solutions random? etc.

Integer solutions: are there $a, b, c \in \mathbb{Z}$ satisfying $a^2 + b^2 = c^2$? Yes!

$$3^2 + 4^2 = 5^2, \quad 5^2 + 12^2 = 13^2, \quad 8^2 + 15^2 = 17^2 \ldots$$

These are called Pythagorean triples.

Trivial solution: $a = b = c = 0$.

(Don't forget to look for the simplest solutions!!)

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Infinitely many?

Claim: If (a, b, c) is a Pythagorean triple, then so is (na, nb, nc) for any $n \in \mathbb{Z}_{\geqslant 0}$. So, yes, but this doesn't generate *all* solutions!

We call a Pythagorean triple (a, b, c) primitive if a, b, and c have no common factors, meaning there is no $d \in \mathbb{Z}_{>0}$ such that a, b, and c are all multiples of d.

Ex: (3,4,5) **Non-ex:** (6,8,10).

Primitive Pythagorean triples (PPTs)

More examples:

$$\begin{array}{cccc} (3,4,5) & (20,21,29) & (28,45,53) & (5,12,13) \\ (9,40,41) & (33,56,65) & (8,15,17) & (7,24,25) \\ (12,35,37) & (11,60,61) & (16,63,65) & (48,55,73) \end{array}$$

Generic even numbers are written $2k$ with $k \in \mathbb{Z}$.
Generic odd numbers are written $2k+1$ with $k \in \mathbb{Z}$.

Square of an even number: $(2k)^2 = 2\underbrace{(2k^2)}_{\text{int}}$ Even!

Square of an odd number:

$$(2k+1)^2 = 4k^2 + 4k + 1 = 2\underbrace{(2k^2 + 2k)}_{\text{integer}} + 1 \quad \text{Odd!}$$

Similarly, you can show that sum of evens and odds follows the pattern

+	even	odd
even	even	odd
odd	odd	even

Primitive Pythagorean triples (PPTs)

You try: For solutions $a, b, c \in ZZ$ to $a^2 + b^2 = c^2$, complete the following table.

a	b	c
even	even	
even	odd	
odd	even	
odd	odd	

Which could possibly be primitive?

Looking for more primitive Pythagorean triples (PPTs)

Without loss of generality (WLOG), assume a is odd and b is even, so c is odd. (WLOG here means that since a and b are interchangeable, we don't need to also consider the cases where the reverse is true)

If $a^2 + b^2 = c^2$, then $a^2 = c^2 - b^2 = (c - b)(c + b)$.

Claims: Assume $a, b, c > 0$.

1. Both $c - b$ and $c + b$ are positive odd integers.
2. There are no divisors common to $c - b$ and $c + b$.
3. Both $c - b$ and $c + b$ are perfect squares.
   Later we will talk about the uniqueness of prime factorizations. For today, assume that if

   $$n = ab \quad \text{for positive integers } n,\ a, \text{ and } b,$$

   then there is a one-to-one correspondence between

   $$\{\text{prime divisors of } n, \text{ counting multiplicity}\} \quad \text{and}$$

   $$\{\text{prime divs of } a, \text{ counting mult}\} \sqcup \{\text{prime divs of } b, \text{ counting mult}\}.$$

Looking for more primitive Pythagorean triples (PPTs)

Let (a, b, c) be a PPT with $a, b, c > 0$. We've shown that $c - b$ and $c + b$ are positive perfect squares with no common divisors.
Define $s, t \in \mathbb{Z}_{>0}$ by

$$c + b = s^2 \quad \text{and} \quad c - b = t^2.$$

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Theorem

Primitive Pythagorean triples are classified by positive integers a, b, and c such that

$$a = st, \quad b = \frac{s^2 - t^2}{2}, \quad \text{and} \quad c = \frac{s^2 + t^2}{2},$$

for odd integers $s > t \geqslant 1$ with no common factors.

You try: Exercise 5, parts (c), (d), and/or (e).