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The result should look like

	A	B	C
1	n	trangle	sqrt
2	1	1	1
3	2	3	1.73205081
4			

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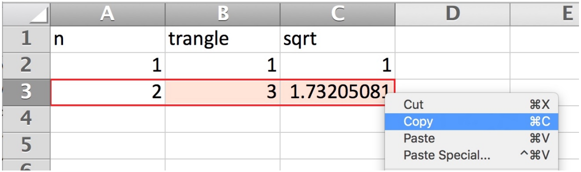

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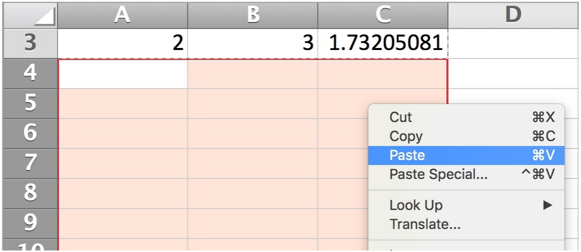

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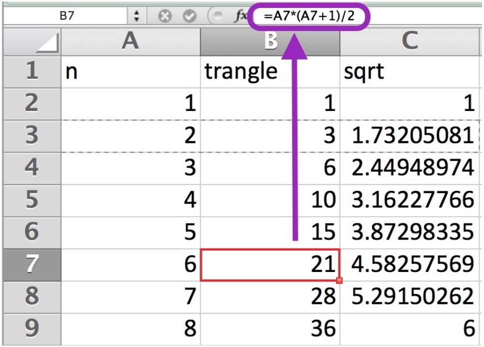

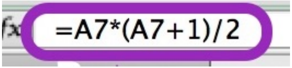

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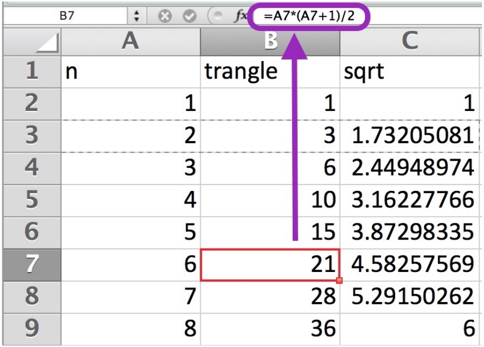

	A	B	C
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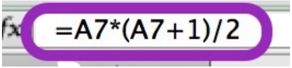
2	1	1	1
3	2	3	1.73205081
4	3	6	2.44948974
5	4	10	3.16227766
6	5	15	3.87298335
7	6	21	4.58257569
8	7	28	5.29150262
9	8	36	6

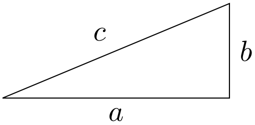


If you *don't* want it to adjust part of a formula with shifts, add a $ in front of the row number, cell number, or both: A$2, $A2, or A2.

Pythagorean Triples

The Pythagorean theorem says the lengths of the sides of a right triangle satisfy the following:

$$a^2 + b^2 = c^2$$



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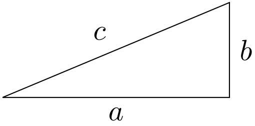
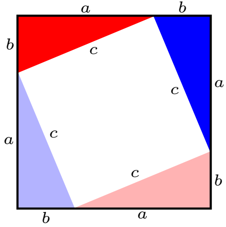


Geometric proof: Compare the area of the white spaces in



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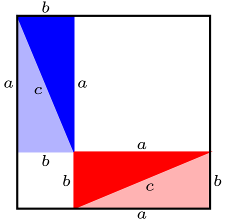

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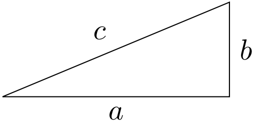

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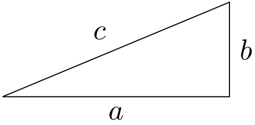

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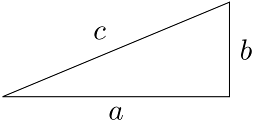

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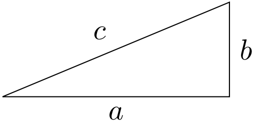

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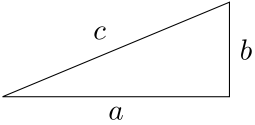

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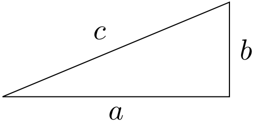

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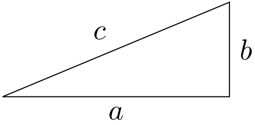

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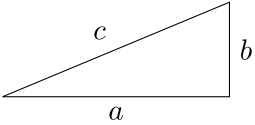

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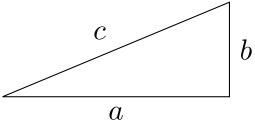

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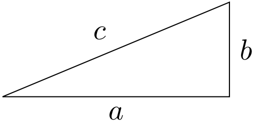

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Ex: (3,4,5) **Non-ex:** (6,8,10).

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More examples:

$$\begin{array}{cccc} (3,4,5) & (20,21,29) & (28,45,53) & (5,12,13) \\ (9,40,41) & (33,56,65) & (8,15,17) & (7,24,25) \\ (12,35,37) & (11,60,61) & (16,63,65) & (48,55,73) \end{array}$$

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$$(2k+1)^2 = 4k^2 + 4k + 1 = 2\underbrace{(2k^2+2k)}_{\text{integer}} + 1 \quad \text{Odd!}$$

Similarly, you can show that sum of evens and odds follows the pattern

+	even	odd
even	even	odd
odd	odd	even

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You try: For solutions $a, b, c \in ZZ$ to $a^2 + b^2 = c^2$, complete the following table.

a	b	c
even	even	
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Without loss of generality (WLOG), assume a is odd and b is even. (WLOG here means that since$a$ and b are interchangeable, we don't need to also consider the cases where the reverse is true)

Looking for more primitive Pythagorean triples (PPTs)

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1. Both $c - b$ and $c + b$ are positive odd integers.
2. There are no divisors common to $c - b$ and $c + b$.
3. Both $c - b$ and $c + b$ are perfect squares.
   Later we will talk about the uniqueness of prime factorizations. For today, assume that if

   $$n = ab \quad \text{for positive integers } n,\ a, \text{ and } b,$$

   then there is a one-to-one correspondence between

   $$\{\text{prime divisors of } n\text{, counting multiplicity}\} \quad \text{and}$$

   $$\{\text{prime divs of } a\text{, counting mult}\} \sqcup \{\text{prime divs of } b\text{, counting mult}\}.$$

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Let (a, b, c) be a PPT with $a, b, c > 0$. We've shown that $c - b$ and $c + b$ are positive perfect squares with no common divisors.

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Theorem

Primitive Pythagorean triples are classified by positive integers a, b, and c such that

$$a = st, \quad b = \frac{s^2 - t^2}{2}, \quad \textit{and} \quad c = \frac{s^2 + t^2}{2},$$

for odd integers $s > t \geqslant 1$ with no common factors.

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You try: Exercise 5, parts (c), (d), and/or (e).