Name 1 | Name 2 | Name 3

A STUDY OF PYTHAGORAS' THEOREM

Instructions: Together in groups of 2 or 3, fill out the following worksheet. You may lift answers from the reading, or answer on your own. Turn in one packet for your group at the end of class.

Theorem (Pythagoras' Theorem)**.** *For a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other sides.*

Part I: Study the theorem statement.

A. Restate the theorem. Restate the theorem as an "If. . . , then. . . " statement, using variable names to be specific, and explicitly identify the assumptions and conclusions.

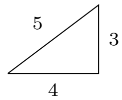
What are the assumptions?

What are the conclusions?

Restatement of the theorem:

B. Draw a picture. Illustrate the assumptions of the theorem with a picture, labeling using the same variable names that you used above.

C. More with pictures and examples

One classic example of a right triangle is 

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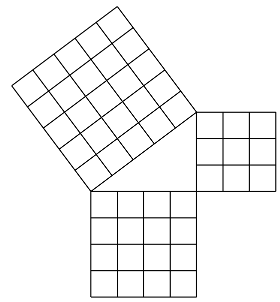
Explain how the following picture illustrates the theorem statement:



Give another example of a right triangle with integer side lengths, and draw the corresponding picture.

Give an example of a right triangle with at least one non-integer-length side.

Give an example of a right triangle with *no* sides of integer length.

D. Apply the theorem to trivial and extreme examples.
What does the theorem statement say about right triangles where one of the legs of the triangle is arbitrarily small (very very close to 0 compared to the other leg)?

E. Rate the strength of the assumptions and conclusions.
Give an example of a slightly weaker statement.

Give an example of related question that would require a slightly stronger statement.

F. Compare with other theorems
Look up and state the "Law of Cosines" and compare the theorem statements.

G. Observe the detail.
What kinds of triangles does this theorem tell you about?

What are some examples of triangles that this theorem doesn't apply to?

H. Classify what the theorem does and how it can be used.
Concretely, what does this theorem enable you to do?

I. Check the converse

State the converse of Pythagoras' Theorem.

Use the Law of Cosines to verify that the converse is true.

Example: Suppose that we have a triangle with sides of length 2, 2.1, and 2.9. We have

$$2^2 + (2.1)^2 = (2.9)^2.$$

The triangle is therefore right-angled.

Question: Did we use Pythagoras' Theorem or its converse to deduce this? Explain.

Example: Suppose that we have a triangle with sides of length 3.6, 7.7, and 8.4. We have

$$(6.6)^2 + (7.7)^2 = (8.4)^2.$$

The triangle is therefore not right-angled.

Question: Did we use Pythagoras' Theorem or its converse to deduce this? Explain.

Part II: Proof of Pythagoras' Theorem.

A. Examples versus proofs. We have have done some examples of Pythagoras' Theorem. Why doesn't this replace *proving* Pythagoras' Theorem?

Explain why the picture in part I.C (with the boxes) is also not a proof of Pythagoras' Theorem.

Now let's consider the following proof of Pythagoras' Theorem.

Proof. The proof can be shown using the two squares in Figure 1. To draw the first square begin by drawing a general right triangle with sides of length a and b, and embedding it in the lower-left corner of a square with side lengths $a+b$. Then rotate the original triangle by $90°$ at a time, placing one copy in each of the other corners of the square to get the left-most picture in Figure 1.

Then we draw another $(a + b) \times (a + b)$ square, and once again place four copies of the original triangle inside this square; but this time, we pair them up to make $a \times b$ and $b \times a$ rectangles, and place them in the upper-left and lower-right corners, respectively. This gives the right-most picture in Figure 1.

Now, we compute the area of the $(a + b) \times (a + b)$ square in two ways. Namely, the area of the left square is

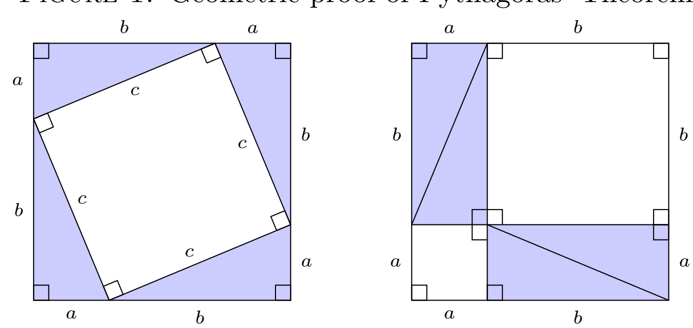
$$c^2 + (4 * \text{Area of } (a, b)\text{-triangle}),$$

and the area of the right square is

$$a^2 + b^2 + (4 * \text{Area of } (a, b)\text{-triangle}).$$

So $c^2 = a^2 + b^2$, as desired. □

FIGURE 1. Geometric proof of Pythagoras' Theorem



Find where the assumptions are used.
Where did we use the fact that the original triangle was a right triangle?

Check the text.
How would the proof had "broken" if the original triangle hadn't been a right triangle?

The "proof" was missing a step! The picture implies that the white interior in the left-most picture in Figure 1 was a square, but this was not actually verified! Use some geometry that the interior angles of that white shape are indeed right angles now.

Check the proof here against the proof given in the book. Can you find other missing steps or assumptions in the book's proof?