A statement is a sentence that is either always true or always false.

Examples

- $2 + 2 = 4$.
- Suppose $x^2 = 2$. Then x is not an integer.
- All cats are gray.
- I have a gray cat.

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Non-examples

- x is an odd number.
- Let "Spot" be a cat. Then Spot is gray.
- This statement is false.

A conditional statement is almost a statement—it *would* be a statement if we added enough extra information. For example

"x is not an integer."

versus

"Suppose $x^2 = 2$. Then x is not an integer."

From the reading:

Which of the following are statements?

(i) Aristotle was Greek.

(ii) Aristotle was great.

(iii) The number $\sqrt{2}$ is rational.

(iv) The square root of an integer is a rational number.

(v) $3x^2 + 20x - 5 = 0$.

(vi) Let x be an integer. Then $\sqrt{x}$ is rational.

(vii) There is a number x such that $\sin(x) = x$.

(viii) There is an infinite number of natural numbers.

(ix) There is an infinite number of rational numbers.

Precision is paramount!

From the reading:

Imagine that there are coins on a table; you count them and find there are exactly three of them—no more, no fewer. Which of the statements are true?

(i) There are four coins on the table.

(ii) There are two coins on the table.

(iii) There are three coins on the table.

(iv) There is a coin on the table.

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Compare this to

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Problem: it is possible for "All cats are gray" and "All cats are not grey" to both be false at the same time.

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Compare this to

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Problem: it is possible for "All cats are gray" and "All cats are not grey" to both be false at the same time.

Notation: If A is a statement,
then we write its negation as $\neg A$ or $\mathrm{not}(A)$, read "not A". (LaTeX: $\neg$ is \neg)

Building new statements from old

Let A and B be statements.

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Example. Let A be the (conditional) statement "x is even" and B be the (conditional) statement "x is a multiple of 3". Then

- $\neg A$ is "x is not a multiple of 2";
- $A \land B$ is "x is a multiple of 6";

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Notation: $\land$ and $\lor$ mimic $\cap$ and $\cup$. Namely, if $\mathcal{A}$ (resp. $\mathcal{B}$) is the set of things satisfying a conditional statement A (resp. B), then

$$\mathcal{A} \cap \mathcal{B} \text{ is the set of things satisfying } A \land B,$$

and

$$\mathcal{A} \cup \mathcal{B} \text{ is the set of things satisfying } A \lor B.$$

Truth tables

Let A and B be statements.

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- $A \wedge B$, read "A and B", is true whenever both A and B are both true.
- $A \vee B$, read "A or B", is true whenever A is true or B is true, or both.

A truth table is just a way of summarizing all the possibilities for statements being true or false.

A	B	$\neg A$	$\neg B$	$A \wedge B$	$A \vee B$
T	T				
T	F				
F	T				
F	F				

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Example: Let's compare $\neg(A \wedge B)$ and $(\neg A) \vee (\neg B)$...

A	B	$A \wedge B$	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$(\neg A) \vee (\neg B)$
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Example: The statement $A \vee \neg A$ is a tautology.

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T	F	F
F	T	F

You try:

Construct truth tables for

(i) $(\neg A) \vee B$,

(ii) $\neg(A \vee B)$,

(iii) $(\neg A) \wedge (\neg B)$,

(iv) $(A \wedge B) \vee (\neg A \vee \neg B)$, and

(v) $(A \wedge B) \wedge (\neg A \vee \neg B)$.

Be sure to show all your steps by including columns for things like $\neg A$, $\neg B$, $A \vee B$, $A \wedge B$, $\neg A \vee \neg B$, and $\neg A \vee \neg B$.

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"If statement A is true, then statement B is true."

Notation: $A \Rightarrow B$ (LaTeX: $\Rightarrow$ is \Rightarrow)

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Examples

(i) If Jill's enrolled in this class, then Jill's a student at CUNY.

(ii) If Jill's a student at CUNY, then Jill's enrolled in this class.

(iii) If $a < b$, then $-a > -b$.

(iv) If $a < b$, then $a^2 < b^2$.

(v) If it's raining outside, then 4 is even.

(vi) If you're enrolled in this class, then 4 is odd.

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Compare this truth table to the one you computed for $(\neg A) \vee B$!

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Other ways of saying "If A then B":

"B if A" and "A only if B".