

# Polya's four-step plan for problem solving

1. Understand the problem.
2. Devise a plan.
3. Execute the plan.
4. Look back.

## Understand the problem.

- ▶ What definitions, notation, etc. do you need to know before you start?
- ▶ Do some work, and make a guess.
  - ▶ Is there a smaller, similar problem that might give you some intuition?
  - ▶ Are there special cases that are easier to understand?
  - ▶ Can you do concrete examples?
  - ▶ Can you draw a picture?
- ▶ Which part of the problem statement is the assumption(s)? Which part is the conclusion(s)? What do you already know about these?  
(Note: Houston calls assumptions “hypotheses”, which is not usually how we use that word. *Hypothesis* usually means a guess based on evidence.)
- ▶ Work backward and forward.
- ▶ Rewrite the problem in an equivalent way.

## Devise a plan.

- ▶ Break into pieces.

*Step 1, step 2, step 3, ...*

*Case 1, case 2, case 3, ...*

- ▶ Find the right level.

Does this problem need a big powerful theorem? Or can you do it just for the definitions?

- ▶ Give things names, so that you can refer to them or use them more easily.

*Let  $A$  be...*

- ▶ Systematically choose a method.

*What standard methods have you already learned? Might any apply? Try one at a time.*

## Devise a plan.

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## Execute the plan.

Do that stuff you planned, checking and reassessing the plan at each step.

## Look back.

- ▶ Sanity check!  
Examples, easy properties, etc.  
Can you find an example contradicting your solution?
- ▶ Revise, revise, revise.  
*Did you use all your assumptions? Did you do things in the best order? Is there an easier/clearer path to your answer?*
- ▶ Reflect.

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**You try:** In your groups, for each of the exercises, do the following:

- (i) In the problem statement, what are the assumptions? What are the conclusions?
- (ii) Do a couple of small examples, or do a similar smaller/easier version of the problem.
- (iii) Solve the problem.