Welcome to Math 308!

Course info

Me: Professor Daugherty, zdaugherty@gmail.com
Website:
https://zdaugherty.ccnysites.cuny.edu/teaching/m308f18/
Textbooks:
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Homework: due on Tuesdays in class, Posted on course website.
FINAL DRAFTS.
Exams: Midterms 10/16&18 and 12/6&11.
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Homework 0: Before class on Tuesday 9/4, send me an email at zdaugherty@gmail.com with subject line "Math 308: Homework 0", answering the questions outlined on the website.

Course expectations

- Read posted sections before class, and bring your own copy of daily notes if needed (posted night before class).
- Come to class, participate, ask questions, work (possibly together) on in-class exercises.
- Come to office hours at least once in the semester. If you can't make my office hour, make an appointment.
- Out of class studying and work should be about 2-3 times the amount of time spent in class (5.5–7 hours/week). Find classmates to study and work with!
- Hand in "final draft" homework, typed up in LaTeX, on time. Get good practice with writing; using words and complete sentences. Ok to work with other people, but write-ups must be your own.
- If there are accessibility accommodations or exam conflicts to be organized, contact me as soon as possible.
- If you send me email, use complete sentences and be specific (ok to send pics of work!).

Definition

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(i) Set containing the numbers 1, 2, and 3 is $\{1, 2, 3\} = \{1, 3, 2\} = \{3, 2, 1\}$. The number 3 is an element of the set, i.e. $3 \in \{1, 2, 3\}$, but $6 \notin \{1, 2, 3\}$.

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Ex: The set $\{1, 2, a, b\}$ has cardinality 4;
the set $\{1, \{2, a, b\}\}$ has cardinality 2.

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The empty set: $\varnothing = \{\}$ (nothing is in here)

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Example: Consider the set $X = \{x, \{x\}\}$. Then $x \in X$ and $\{x\} \subsetneq X$, but <u>also</u> we have $\{x\} \in X$.

Operations on sets

Let X and Y be sets.
The union of X and Y is

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The product of X and Y is

$$X \times Y = \{(x, y) \mid x \in X, y \in Y\},$$

the set of all possible (ordered) pairs (x, y) where $x \in X$ and $y \in Y$.

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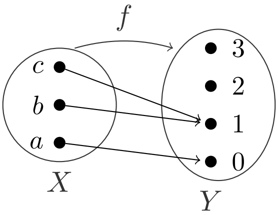

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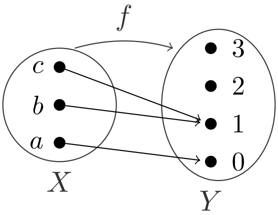

Note: every element of X gets one element in Y, but not necessarily vice versa; and two distinct elements of X may map to the same element in Y.

Some examples:

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Non-example: The formula $f(x) = 1/(1-x)$ does not define a function from $\mathbb{R}$ to $\mathbb{R}$, since it is not defined at $x = 1$.

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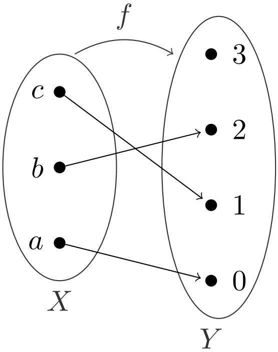

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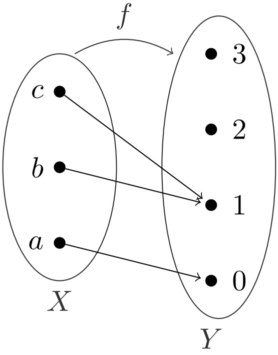

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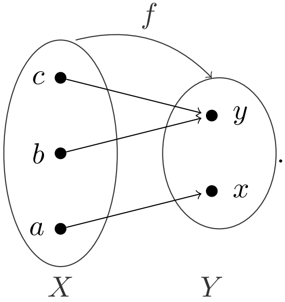

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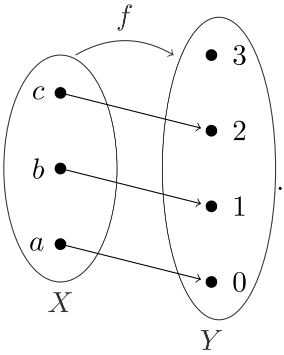

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