YOUR NAME HERE
Math 308
Homework 6
10/9/2018

1. Using what we learned from chapters 10 & 11, rewrite you proofs of the following problems.
   (a) Let a and b be real numbers. Show that
   $$\frac{a+b}{2} \geq \sqrt{ab} \qquad \text{for all} \quad 0 < a \leq b.$$
   (b) Show that
   $$a^2 + b^2 + c^2 \geq ab + bc + ca$$
   for all positive integers a, b, and c.
2. Submit a revision of your Proof Without Words from Homework 2.
3. Use a truth table to check that for statements A, B, and C, we have
   $$\big((A \Rightarrow B) \wedge (B \Rightarrow C) \wedge (C \Rightarrow A)\big)$$
   $$\Leftrightarrow$$
   $$\big((A \Leftrightarrow B) \wedge (B \Leftrightarrow C) \wedge (A \Leftrightarrow C)\big)$$
   is a tautology. (You might want to transpose your truth table, so that the columns are the T/F options, and the rows are the statements.)
4. Show that the sum of two consecutive odd numbers is a multiple of 4.
5. What is wrong with the following argument?

   We have
   $$2 = 4 \Rightarrow 2\pi = 4\pi \Rightarrow \sin(2\pi) = \sin(4\pi) \Rightarrow 0 = 0.$$
   Therefore $2 = 4$.
6. Consider the following:

   Claim. For all $n \in \mathbb{N}$, the number $n^2 + 5n + 6$ is not prime.

   Find the mistake in the following 'proofs'.
   (a)
   For $n = 2$, *we have* $n^2 + 5n + 6 = 22 + 10 + 6 = 20$, *which is not prime. Hence the theorem is true.*
   (b)
   Suppose that n *is a natural number, so in particular* $n > 0$. *If* $n^2 + 5n + 6$ *is not prime, then* $n^2 + 5n + 6 = pq$ *for some natural numbers* p *and* q *such that* $0 < p < n^2 + 5n + 6$ *and* $0 < q < n^2 + 5n + 6$. *Since* $n^2 + 5n + 6 = pq$ *and* p *and* q *do not equal* 1 *or* $n^2 + 5n + 6$, *then* $n^2 + 5n + 6$ *is not prime, which was to be shown.*
7. Find the error in the following proof that the sum of two rational numbers is rational.

   Suppose that m *and* n *are rational numbers, so that we can write* $m = p/q$ *and* $n = r/s$ *where* $p, q, r,$ *and* s *are integers (and* q *and* s *are non-zero). Then*
   $$m + n = \frac{p}{q} + \frac{r}{s}.$$
   Since the sum of two fractions is a fraction, then $m + n$ *must be a fraction. Hence,* $m + n$ *is rational.*

8. Heres another error that people make. What is it? ??

    The following proves the statement $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \left((y+1)x^2 = \frac{1+y}{y^4}\right)$*:*

    Suppose that $x = 1/y^2$*. Then*

$$(y+1)x^2 = (y+1)\left(\frac{1}{y^2}\right)^2 = \frac{y+1}{y^4} = \frac{1+y}{y^4}.$$

9. Write a basic plan/outline for a proof for each of the *Proof Lab: Direct Proof* problems.

    For example, suppose you had the following problem:

    Let x *and* y *be real numbers. If* $y > x > 0$*, then*

$$\frac{x+1}{y+1} > \frac{x}{y}.$$

Then your basic outline would be:

Proof sketch.

- **Start with:** "Let x and y be real numbers with $y > x > 0$."
- **Goal:** Conclude $\frac{x+1}{y+1} > \frac{x}{y}$.
- **To try:** Start with $\frac{x+1}{y+1} > \frac{x}{y}$ and work backwards? Don't forget to reorder computations in the end!