HOMEWORK 5
MATH 308
DUE: 10/2/2018

1. The connective "unless" can be ambiguous, and this exercise will pinpoint the ambiguity.

   We awake at dawn, and we are told

   $$\text{“}\underbrace{\textit{We will have a picnic today}}_{A} \quad \text{unless} \quad \underbrace{\textit{it is raining at 10 A.M.}}_{B}\text{”}$$

   Let $A \star B$ denote A unless B (not a standard notation). Complete as much of a truth table as possible for$A \star B$, and discuss any ambiguous lines.
2. Use a truth tables to check that the following are tautologies.
   (a) $\big((A \Rightarrow B) \wedge (B \Rightarrow C)\big) \Rightarrow (A \Rightarrow C)$.
   (b) $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$.
3. Rewrite the following as "if. . . , then. . . " statements.
   (a) A sufficient condition for Peter to win the Championship is that he wins in Brazil.
   (b) To be President of the US it is necessary to be born in the US.
   (c) To be Prime Minister of India it is not necessary to be born in India.
   (d) You can't make an omelette without breaking some eggs.
   (e) A masters degree is required for this position.
   (f) To cross this river, we just need to find a boat. (Careful!)
4. Let A be the statement "$x^2 - 2x - 3 > 0$" and B be the statement "$x > 3$". Which of the following are true, which are false? Be sure to justify your overall answer.

   $$A \Rightarrow B, \qquad B \Rightarrow A, \qquad \neg A \Rightarrow \neg B, \qquad \neg B \Rightarrow \neg A,$$
   $$\text{“}A \text{ is necessary for } B\text{”}, \qquad \text{“}B \text{ is necessary for } A\text{”},$$
   $$\text{“}A \text{ is sufficient for } B\text{”}, \quad \text{and} \quad \text{“}B \text{ is sufficient for } A\text{”}.$$

5. Which of the following statements are equivalent? For each, also give write the sentence in terms of logical notation, with A being the statement "my team won the last game" and B the statement "my team won the championship". (Assume ties are not an option, so that "didn't win" is the same thing as "lost".)
   (a) If my team lost the last game, then they must have lost the championship.
   (b) If my team lost the last game, then your team won the championship.
   (c) If my team lost the last game, then they won the championship.
   (d) If my team won the championship, then they won the last game.
   (e) If my team won the last game, then they won the championship.
   (f) If my team lost the championship, then they must have lost the last game.
6. What are the negation, inverse, converse, and contrapositive of each of the following?
   (a) If $x > 5$, then X is red.
   (b) Taking a shower is necessary for me to be happy all day.
   (c) It isn't necessary to understand things to argue about them.
   (d) Stop doing that, or I'll get angry.
   (e) $A \Rightarrow (B \Rightarrow C)$ (Simplify your answer.)

7. Let a and b be integers. Prove the following.
   [*You may take for granted that if a is even, then $a = 2k$ for some integer k; that if a is odd then $a = 2\ell + 1$ for some integer ℓ; and that no number can be both even and odd.*]
   (a) We have a is odd if and only if $a + 2$ is odd.
   (b) We have a is odd if and only if a^3 is odd.
   (c) We have ab is odd if and only if a and b are both odd.
8. Rewrite the following using $\forall$ and $\exists$.
   (a) For all integers x, we have x is odd or even.
   (Your answer should include a definition of even/odd using $\exists$.)
   (b) There exist two positive numbers such that their sum is negative.
9. Consider the statement "If a and b are real numbers with $a \neq 0$, then $ax + b = 0$ has a solution."
   (a) Rewrite this statement using symbolic notation $\forall$ and $\exists$.
   (b) Negate this statement, giving your answer both in symbolic notation, and in words.
10. Negate the following.
   (a) There exists a grey cat.
   (b) Every cat has an owner.
   (c) Some of the students in the class are not here today.
   (d) For all $x, y \in \mathbb{Z}_{>0}$ there exists $z \in \mathbb{Z}_{>0}$ such that $x = y + z$.
   (e) The number $\sqrt{x}$ is rational if x is an integer.
11. For each of the following,
   (i) restate in words;
   (ii) decide whether it's true or false; and
   (iii) prove or disprove accordingly.
   (a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}(x^2 = y)$
   (b) $\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}(x^2 = y)$
   (c) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}(x^2 = y)$
   (d) $\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}(x^2 = y)$