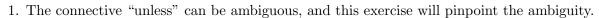
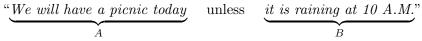
## **HOMEWORK 5 MATH 308**

DUE: 10/2/2018



We awake at dawn, and we are told



Let  $A \star B$  denote A unless B (not a standard notation). Complete as much of a truth table as possible for  $A \star B$ , and discuss any ambiguous lines.

- 2. Use a truth tables to check that the following are tautologies.
  - (a)  $((A \Rightarrow B) \land (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$ .
  - (b)  $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$ .
- 3. Rewrite the following as "if..., then..." statements.
  - (a) A sufficient condition for Peter to win the Championship is that he wins in Brazil.
  - (b) To be President of the US it is necessary to be born in the US.
  - (c) To be Prime Minister of India it is not necessary to be born in India.
  - (d) You can't make an omelette without breaking some eggs.
  - (e) A masters degree is required for this position.
  - (f) To cross this river, we just need to find a boat. (Careful!)
- 4. Let A be the statement " $x^2 2x 3 > 0$ " and B be the statement "x > 3". Which of the following are true, which are false? Be sure to justify your overall answer.

$$A\Rightarrow B, \qquad B\Rightarrow A, \qquad \neg A\Rightarrow \neg B, \qquad \neg B\Rightarrow \neg A,$$
 "A is necessary for B", "B is necessary for A",

"A is sufficient for B", and "B is sufficient for A".

- 5. Which of the following statements are equivalent? For each, also give write the sentence in terms of logical notation, with A being the statement "my team won the last game" and B the statement "my team won the championship". (Assume ties are not an option, so that "didn't win" is the same thing as "lost".)
  - (a) If my team lost the last game, then they must have lost the championship.
  - (b) If my team lost the last game, then your team won the championship.
  - (c) If my team lost the last game, then they won the championship.
  - (d) If my team won the championship, then they won the last game.
  - (e) If my team won the last game, then they won the championship.
  - (f) If my team lost the championship, then they must have lost the last game.
- 6. What are the negation, inverse, converse, and contrapositive of each of the following?
  - (a) If x > 5, then X is red.
  - (b) Taking a shower is necessary for me to be happy all day.
  - (c) It isn't necessary to understand things to argue about them.
  - (d) Stop doing that, or I'll get angry.
  - (e)  $A \Rightarrow (B \Rightarrow C)$  (Simplify your answer.)

2 HOMEWORK 5

7. Let a and b be integers. Prove the following.

[You may take for granted that if a is even, then a = 2k for some integer k; that if a is odd then  $a = 2\ell + 1$  for some integer  $\ell$ ; and that no number can be both even and odd.]

- (a) We have a is odd if and only if a + 2 is odd.
- (b) We have a is odd if and only if  $a^3$  is odd.
- (c) We have ab is odd if and only if a and b are both odd.
- 8. Rewrite the following using  $\forall$  and  $\exists$ .
  - (a) For all integers x, we have x is odd or even.

(Your answer should include a definition of even/odd using  $\exists$ .)

- (b) There exist two positive numbers such that their sum is negative.
- 9. Consider the statement "If a and b are real numbers with  $a \neq 0$ , then ax + b = 0 has a solution."
  - (a) Rewrite this statement using symbolic notation  $\forall$  and  $\exists$ .
  - (b) Negate this statement, giving your answer both in symbolic notation, and in words.
- 10. Negate the following.
  - (a) There exists a grey cat.
  - (b) Every cat has an owner.
  - (c) Some of the students in the class are not here today.
  - (d) For all  $x, y \in \mathbb{Z}_{>0}$  there exists  $z \in \mathbb{Z}_{>0}$  such that x = y + z.
  - (e) The number  $\sqrt{x}$  is rational if x is an integer.
- 11. For each of the following,
  - (i) restate in words;
  - (ii) decide whether it's true or false; and
  - (iii) prove or disprove accordingly.
  - (a)  $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}(x^2 = y)$
  - (b)  $\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}(x^2 = y)$
  - (c)  $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}(x^2 = y)$
  - (d)  $\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}(x^2 = y)$