

**HOMEWORK 4**  
**MATH 308**  
**DUE: 9/27/2018**

1. Which of the following are statements? Of the sentences that are statements, which are true and which are false?
  - (a) Life is sweet.
  - (b) Is 2 a prime?
  - (c) Prove that 2 is a prime.
  - (d) The President of the United States in 1789 was a man.
  - (e) The President of the United States in 2089 will be a woman.

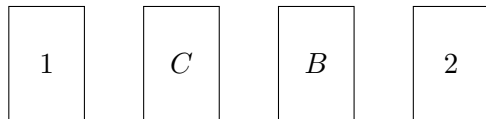
2. Let  $A$  be the statement “Howard fell” and let  $B$  be the statement “Howard broke his leg.”

- (a) Write English statements corresponding to each of these:

$$A \wedge B, \quad \neg A \wedge \neg B, \quad \neg(A \wedge B), \quad B \wedge \neg A.$$

- (b) For each of the statements in (a), under what circumstances would that statement be false? (Again, in sentences, using the example of Howard.)

3. Suppose four cards are given, each of which has a letter on one side and a number on the other. They are displayed as follows:



Which card(s) need not be turned over in order to determine the truth value of the statement, “If a card has  $B$  on one side, then it has 2 on the other side”?

4. Represent the following two statements as implications.
  - (a) I go to the movies in the afternoon if it is rainy.
  - (b) I go to the movies in the afternoon only if it is rainy.
5. Suppose that  $A$  is true and  $B$  is false. Which of the following are true?
  - (a)  $A \Rightarrow B$
  - (b)  $B \Rightarrow A$
  - (c)  $\neg B \Rightarrow A$
  - (d)  $A \Rightarrow A$
  - (e)  $A \vee \neg B$
  - (f)  $\neg A \Rightarrow A$
  - (g)  $\neg(A \vee B)$
6. Construct truth tables for the following, and decide whether each is a tautology, contradiction, or neither. Be sure to include all the intermediate steps when constructing your truth tables!
  - (a)  $\neg(A \vee \neg B)$
  - (b)  $A \Rightarrow (B \vee \neg A)$
  - (c)  $B \wedge \neg(\neg A) \Rightarrow B$

(d)  $((\neg A \Rightarrow B) \wedge (\neg A \Rightarrow \neg B)) \Rightarrow A$

7. Show that each of the following are equivalent statements using truth tables. (Your truth tables will need  $2^3 = 8$  rows.)
- (a)  $A \vee (B \vee C)$  and  $(A \vee B) \vee C$  (showing that  $A \vee B \vee C$  is well-defined).
  - (b)  $A \wedge (B \vee C)$  and  $(A \wedge B) \vee (A \wedge C)$ .
8. Negate the following statements (giving your answers in sentences).
- (a)  $A$  is true or  $B$  is false.
  - (b)  $A$  is false and  $B$  is true.
  - (c)  $A$  is true or  $B$  is true.