

HOMEWORK 3
MATH 308
DUE: 9/20/2018

1. For each of (a)–(f), do the following:

- (i) In the problem statement, what are the assumptions? What are the conclusions?
- (ii) Do a couple of small examples, or do a similar smaller/easier version of the problem.
- (iii) Solve the problem. As always, your writeup should use the writing skills you developed in Chapters 3–4.

[Hint: Any time you see a root (square, seventh, whatever), try to get rid of it.]

(a) Show that

$$\frac{a+b}{2} \geq \sqrt{ab} \quad \text{for all } 0 < a \leq b.$$

(b) Show that

$$a^2 + b^2 + c^2 \geq ab + bc + ca$$

for all positive integers a , b , and c .

(c) Let $f(x) = 1/(1-x)$. Define the function f^r by

$$f^r(x) := \underbrace{f(f(\cdots(f(f(x))))\cdots)}_{r \text{ times}}.$$

Find $f^{653}(56)$.

(d) Without using a calculator, show that $\sqrt[7]{7!} < \sqrt[8]{8!}$.

(e) Without using a calculator, show that $\sqrt{100001} - \sqrt{100000} < \frac{1}{2\sqrt{100000}}$.

(f) Bottle A contains a liter of milk and bottle B contains a liter of coffee. A spoonful of coffee from B is poured into A , and the contents are mixed well. Liquid from A is then poured into B until B has one liter of liquid. Is the fraction of coffee in A greater than the fraction of milk in B , or vice versa?

[Hint: Giving things names will be super important here! You may also assume that your spoon isn't giant.]