HOMEWORK 1
MATH 308
DUE: 9/4/2018

1. What is the cardinality of the following sets?
   (a) $\{1, 2, a, x, \text{apple}\}$
   (b) $\{1, 2, \{a, x\}, \text{apple}\}$
   (c) $\{\{1, 2, \{a, x\}\}, \text{apple}\}$
   (d) $\{\{1, 2, \{a, x\}, \text{apple}\}\}$
   (e) $\emptyset$
   (f) $\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

2. Describe each of the following sets in the format $\{\underbrace{\qquad}_{\text{objects}} | \underbrace{\qquad}_{\text{conditions}}\}$:
   (a) $A = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, \dots\}$;
   (b) $B = \{3, 5, 6, 9, 10, 12, 15, 18, \dots\}$;
   (c) $D = \{1, 1/2, 1/3, 1/4, 1/5, \dots\}$.

3. Find $X \cup Y$ and $X \cap Y$ for the following:
   (a) $X = \{0, 2, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$;
   (b) $X = \{-1, 2, 17\}$ and $Y = \{1, 3, 5, 7, 9\}$;
   (c) $X = \mathbb{Q}$ and $Y = \{0, 1, \pi, 1/2\}$;
   (d) $X = \{x \in \mathbb{R} \mid x > 7\}$ and $Y = \{y \in \mathbb{N} \mid x > 5\}$.

4. Find $\mathbb{N} \cap \mathbb{N}$, $\mathbb{N} \cap \emptyset$, and $\mathbb{N} \cap \mathbb{Q}$.

5. Let $X = \{x \in \mathbb{Z} \mid 0 \leq x \leq 10\}$, and A and B be the subsets of X given by $A = \{0, 2, 3, 9\}$ and $B = \{x \in X \mid 5 \leq x < 9\}$. Find $A \cap B$, $A \cup B$, $A - B$, $B - A$, $A \times B$, A^c, and B^c.

6. In each part give an example of sets A, B, C that simultaneously satisfy the stated conditions:
   (a) $A \in B$, $B \in C$, $A \notin C$;
   (b) $A \in B$, $B \in C$, $A \in C$;
   (c) $A \notin B$, $B \in C$, $A \in C$.

7. Suppose that A, B and C are subsets of a set X. Use examples to investigate the following sets. In each case, make a hypothesis.
   (a) $(A \cap B) \cup (A \cap C)$ versus $A \cap (B \cup C)$;
   (b) $(A \cup B)^c$ versus $A^c \cap B^c$;
   (c) $(A \cap B)^c$ versus $A^c \cap B^c$.

8. Let S and T be sets of three elements. How many functions are there from S to T?

9. Which of the following functions are injective? Which are surjective?
   (a) The constant function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3$.
   (b) The constant function $f : \mathbb{R} \to \{3\}$ defined by $f(x) = 3$.
   (c) The identity function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x$.
   (d) The function $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(x) = x$.
   (e) The cardinality function, given by $|| : \text{Finite sets} \to \mathbb{Z}_{\geq 0}$.

10. Analyze how you approached the reading of Chapter 1 of "How to Think Like a Mathematician".
   (a) If you had not met the material in this chapter before, then did you attempt to understand everything?

(b) If you had encountered the material before, did you check to see that the author had not made any mistakes? Did you compare how the books definitions agreed with or differed from those your had learned before?