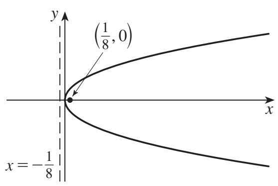
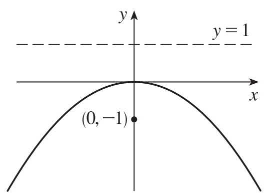
ANSWERS

S Click here for solutions.

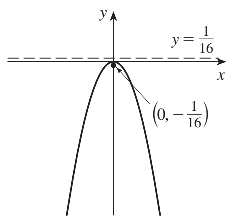
1. $(0, 0)$, $\left(\frac{1}{8}, 0\right)$, $x = -\frac{1}{8}$



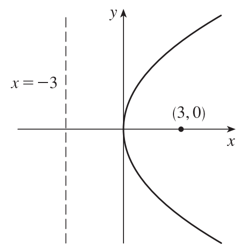
2. $(0, 0)$, $(0, -1)$, $y = 1$



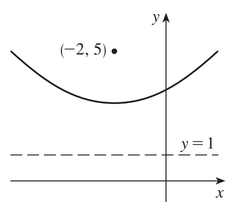
3. $(0, 0)$, $\left(0, -\frac{1}{16}\right)$, $y = \frac{1}{16}$



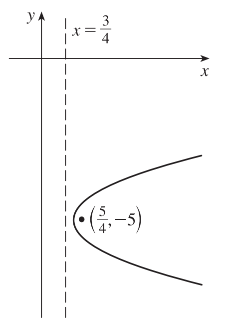
4. $(0, 0)$, $(3, 0)$, $x = -3$



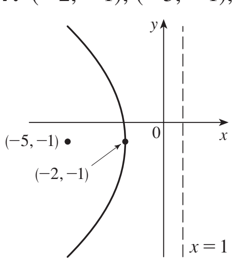
5. $(-2, 3)$, $(-2, 5)$, $y = 1$



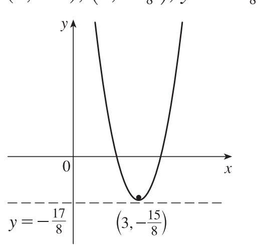
6. $(1, -5)$, $\left(\frac{5}{4}, -5\right)$, $x = \frac{3}{4}$



7. $(-2, -1)$, $(-5, -1)$, $x = 1$



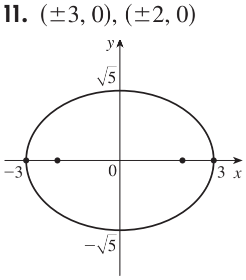
8. $(3, -2)$, $\left(3, -\frac{15}{8}\right)$, $y = -\frac{17}{8}$



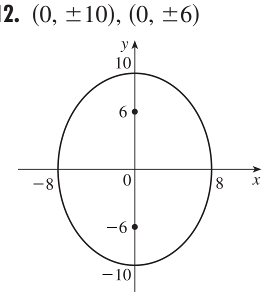
9. $x = -y^2$, focus $\left(-\frac{1}{4}, 0\right)$, directrix $x = \frac{1}{4}$

10. $(x - 2)^2 = 2(y + 2)$, focus $\left(2, -\frac{3}{2}\right)$, directrix $y = -\frac{5}{2}$

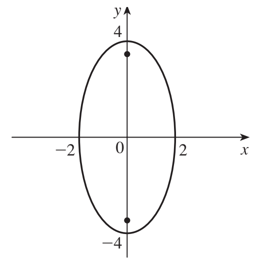
11. $(\pm 3, 0)$, $(\pm 2, 0)$



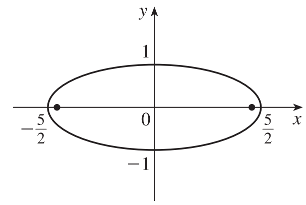
12. $(0, \pm 10)$, $(0, \pm 6)$



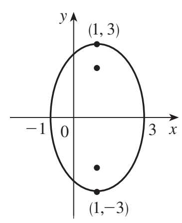
13. $(0, \pm 4)$, $\left(0, \pm 2\sqrt{3}\right)$



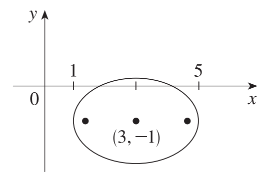
14. $\left(\pm\frac{5}{2}, 0\right)$, $\left(\pm\frac{\sqrt{21}}{2}, 0\right)$



15. $(1, \pm 3)$, $\left(1, \pm\sqrt{5}\right)$



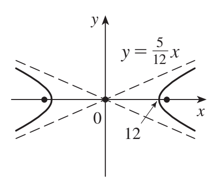
16. $(1, -1)$ and $(5, -1)$, $\left(3 \pm \sqrt{2}, -1\right)$



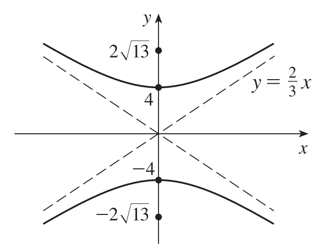
17. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, foci $\left(0, \pm\sqrt{5}\right)$

18. $\dfrac{(x - 2)^2}{9} + \dfrac{(y - 1)^2}{4} = 1$, foci $\left(2 \pm \sqrt{5}, 1\right)$

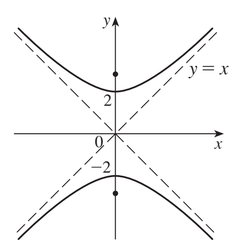
19. $(\pm 12, 0)$, $(\pm 13, 0)$, $y = \pm\frac{5}{12}x$



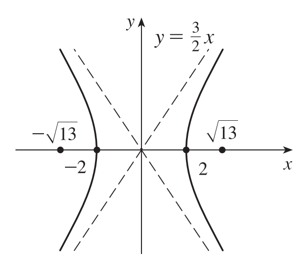
20. $(0, \pm 4)$, $\left(0, \pm 2\sqrt{13}\right)$, $y = \pm\frac{2}{3}x$



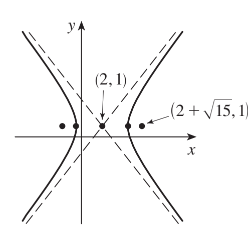
21. $(0, \pm 2)$, $\left(0, \pm 2\sqrt{2}\right)$, $y = \pm x$



22. $(\pm 2, 0)$, $\left(\pm\sqrt{13}, 0\right)$, $y = \pm\frac{3}{2}x$



23. $\left(2 \pm \sqrt{6}, 1\right)$, $\left(2 \pm \sqrt{15}, 1\right)$, $y - 1 = \pm\left(\sqrt{6}/2\right)(x - 2)$



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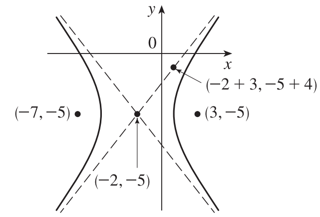
24. $(-5, -5)$ and $(1, -5)$, $(-7, -5)$ and $(3, -5)$, $y + 5 = \pm\frac{4}{3}(x + 2)$



25. Parabola, $(0, -1)$, $\left(0, -\frac{3}{4}\right)$ **26.** Hyperbola, $(\pm 1, 0)$, $\left(\pm\sqrt{2}, 0\right)$

27. Ellipse, $\left(\pm\sqrt{2}, 1\right)$, $(\pm 1, 1)$ **28.** Parabola, $(0, 4)$, $\left(\frac{3}{2}, 4\right)$

29. Hyperbola, $(0, 1)$, $(0, -3)$; $\left(0, -1 \pm \sqrt{5}\right)$

30. Ellipse, $\left(-\frac{1}{2}, \pm 1\right)$, $\left(-\frac{1}{2}, \pm\sqrt{3}/2\right)$

31. $x^2 = -8y$ **32.** $y^2 = 24(x - 1)$ **33.** $y^2 = -12(x + 1)$

34. $(x - 3)^2 = 16(y - 2)$ **35.** $y^2 = 16x$

36. $2x^2 + 4x - y + 3 = 0$ **37.** $\dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$

38. $\dfrac{x^2}{144} + \dfrac{y^2}{169} = 1$ **39.** $\dfrac{x^2}{12} + \dfrac{(y - 4)^2}{16} = 1$

40. $\dfrac{(x - 4)^2}{25} + \dfrac{(y + 1)^2}{9} = 1$ **41.** $\dfrac{(x - 2)^2}{9} + \dfrac{(y - 2)^2}{5} = 1$

42. $\dfrac{2x^2}{9 + \sqrt{17}} + \dfrac{2y^2}{1 + \sqrt{17}} = 1$

43. $y^2 - \frac{1}{8}x^2 = 1$ **44.** $\frac{1}{16}x^2 - \frac{1}{20}y^2 = 1$

45. $\dfrac{(x - 4)^2}{4} - \dfrac{(y - 3)^2}{5} = 1$

46. $\frac{1}{9}(y - 3)^2 - \frac{1}{16}(x - 2)^2 = 1$ **47.** $\frac{1}{9}x^2 - \frac{1}{36}y^2 = 1$

48. $\frac{1}{2}(x - 4)^2 - \frac{1}{2}(y - 2)^2 = 1$

49. $\dfrac{x^2}{3{,}763{,}600} + \dfrac{y^2}{3{,}753{,}196} = 1$

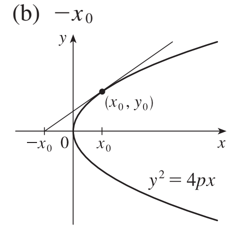
50. (a) $p = \frac{5}{2}$, $y^2 = 10x$ (b) $2\sqrt{110}$

51. (a) $\dfrac{121x^2}{1{,}500{,}625} - \dfrac{121y^2}{3{,}339{,}375} = 1$ (b) ≈ 248 mi

54. $3x^2 - 2xy + 3y^2 = 8$

55. (a) Ellipse (b) Hyperbola (c) No curve

56. (b) $-x_0$ **57.** 9.69 **58.** 3.64×10^{10} km



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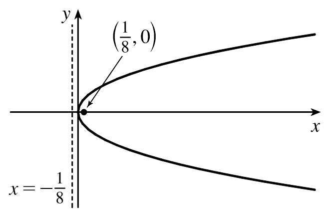
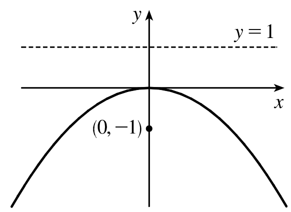
SOLUTIONS

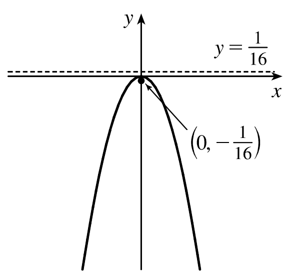
1. $x = 2y^2 \quad\Rightarrow\quad y^2 = \frac{1}{2}x$. $4p = \frac{1}{2}$, so $p = \frac{1}{8}$. The vertex is $(0, 0)$, the focus is $\left(\frac{1}{8}, 0\right)$, and the directrix is $x = -\frac{1}{8}$.



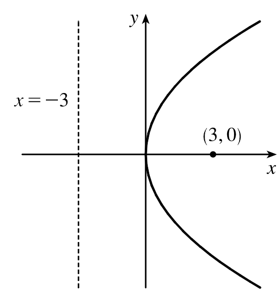
2. $4y + x^2 = 0 \quad\Rightarrow\quad x^2 = -4y$. $4p = -4$, so $p = -1$. The vertex is $(0, 0)$, the focus is $(0, -1)$, and the directrix is $y = 1$.



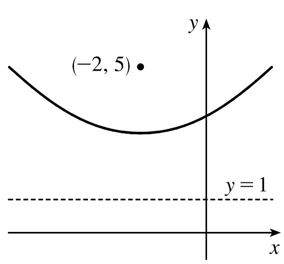
3. $4x^2 = -y \quad\Rightarrow\quad x^2 = -\frac{1}{4}y$. $4p = -\frac{1}{4}$, so $p = -\frac{1}{16}$. The vertex is $(0, 0)$, the focus is $\left(0, -\frac{1}{16}\right)$, and the directrix is $y = \frac{1}{16}$.



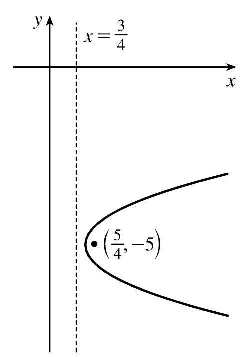
4. $y^2 = 12x$. $4p = 12$, so $p = 3$. The vertex is $(0, 0)$, the focus is $(3, 0)$, and the directrix is $x = -3$.



5. $(x + 2)^2 = 8(y - 3)$. $4p = 8$, so $p = 2$. The vertex is $(-2, 3)$, the focus is $(-2, 5)$, and the directrix is $y = 1$.



6. $x - 1 = (y + 5)^2$. $4p = 1$, so $p = \frac{1}{4}$. The vertex is $(1, -5)$, the focus is $\left(\frac{5}{4}, -5\right)$, and the directrix is $x = \frac{3}{4}$.



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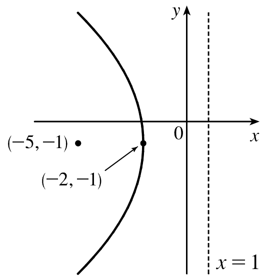
7. $y^2 + 2y + 12x + 25 = 0 \quad \Rightarrow$

$y^2 + 2y + 1 = -12x - 24 \quad \Rightarrow$

$(y+1)^2 = -12(x+2)$. $4p = -12$, so $p = -3$.

The vertex is $(-2, -1)$, the focus is $(-5, -1)$, and the directrix is $x = 1$.



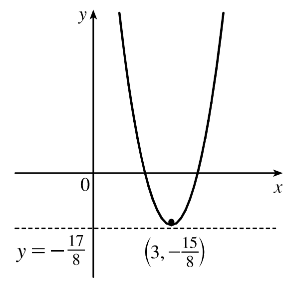
8. $y + 12x - 2x^2 = 16 \quad \Rightarrow$

$2x^2 - 12x = y - 16 \quad \Rightarrow$

$2(x^2 - 6x + 9) = y - 16 + 18 \quad \Rightarrow$

$2(x-3)^2 = y + 2 \quad \Rightarrow \quad (x-3)^2 = \frac{1}{2}(y+2)$.

$4p = \frac{1}{2}$, so $p = \frac{1}{8}$. The vertex is $(3, -2)$, the focus is $\left(3, -\frac{15}{8}\right)$, and the directrix is $y = -\frac{17}{8}$.

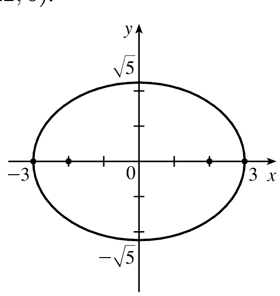


9. The equation has the form $y^2 = 4px$, where $p < 0$. Since the parabola passes through $(-1, 1)$, we have $1^2 = 4p(-1)$, so $4p = -1$ and an equation is $y^2 = -x$ or $x = -y^2$. $4p = -1$, so $p = -\frac{1}{4}$ and the focus is $\left(-\frac{1}{4}, 0\right)$ while the directrix is $x = \frac{1}{4}$.

10. The vertex is $(2, -2)$, so the equation is of the form $(x-2)^2 = 4p(y+2)$, where $p > 0$. The point $(0, 0)$ is on the parabola, so $4 = 4p(2)$ and $4p = 2$. Thus, an equation is $(x-2)^2 = 2(y+2)$. $4p = 2$, so $p = \frac{1}{2}$ and the focus is $\left(2, -\frac{3}{2}\right)$ while the directrix is $y = -\frac{5}{2}$.

11. $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1 \quad \Rightarrow \quad a = \sqrt{9} = 3, b = \sqrt{5}$,

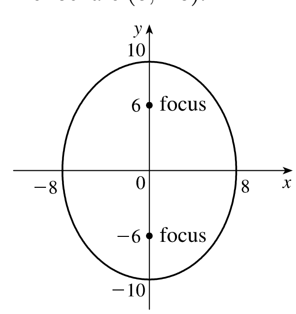
$c = \sqrt{a^2 - b^2} = \sqrt{9 - 5} = 2$. The ellipse is centered at $(0, 0)$, with vertices at $(\pm 3, 0)$. The foci are $(\pm 2, 0)$.



12. $\dfrac{x^2}{64} + \dfrac{y^2}{100} = 1 \quad \Rightarrow \quad a = \sqrt{100} = 10$,

$b = \sqrt{64} = 8$, $c = \sqrt{a^2 - b^2} = \sqrt{100 - 64} = 6$.

The ellipse is centered at $(0, 0)$, with vertices at $(0, \pm 10)$. The foci are $(0, \pm 6)$.



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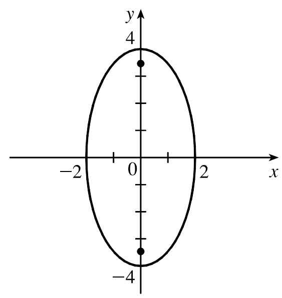
13. $4x^2 + y^2 = 16 \quad\Rightarrow\quad \dfrac{x^2}{4} + \dfrac{y^2}{16} = 1 \quad\Rightarrow$

$a = \sqrt{16} = 4$, $b = \sqrt{4} = 2$,

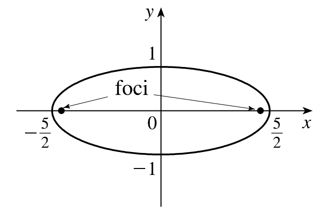
$c = \sqrt{a^2 - b^2} = \sqrt{16 - 4} = 2\sqrt{3}$. The ellipse is centered at $(0, 0)$, with vertices at $(0, \pm 4)$. The foci are $\left(0, \pm 2\sqrt{3}\right)$.



14. $4x^2 + 25y^2 = 25 \quad\Rightarrow\quad \dfrac{x^2}{25/4} + \dfrac{y^2}{1} = 1 \quad\Rightarrow$

$a = \sqrt{\frac{25}{4}} = \frac{5}{2}$, $b = \sqrt{1} = 1$,

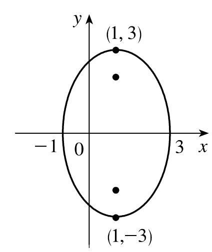
$c = \sqrt{a^2 - b^2} = \sqrt{\frac{25}{4} - 1} = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$. The ellipse is centered at $(0, 0)$, with vertices at $\left(\pm\frac{5}{2}, 0\right)$. The foci are $\left(\pm\frac{\sqrt{21}}{2}, 0\right)$.



15. $9x^2 - 18x + 4y^2 = 27 \quad\Leftrightarrow\quad 9(x^2 - 2x + 1) + 4y^2 = 27 + 9 \quad\Leftrightarrow$

$9(x - 1)^2 + 4y^2 = 36 \quad\Leftrightarrow\quad \dfrac{(x - 1)^2}{4} + \dfrac{y^2}{9} = 1 \quad\Rightarrow\quad a = 3, b = 2,$

$c = \sqrt{5} \quad\Rightarrow\quad$ center $(1, 0)$, vertices $(1, \pm 3)$, foci $\left(1, \pm\sqrt{5}\right)$

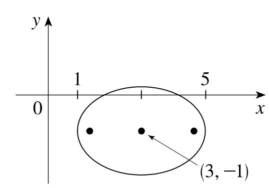


16. $x^2 - 6x + 2y^2 + 4y = -7 \quad\Leftrightarrow$

$x^2 - 6x + 9 + 2(y^2 + 2y + 1) = -7 + 9 + 2 \quad\Leftrightarrow$

$(x - 3)^2 + 2(y + 1)^2 = 4 \quad\Leftrightarrow$

$\dfrac{(x - 3)^2}{4} + \dfrac{(y + 1)^2}{2} = 1 \quad\Rightarrow\quad a = 2, b = \sqrt{2} = c \quad\Rightarrow\quad$ center $(3, -1)$, vertices $(1, -1)$ and $(5, -1)$, foci $\left(3 \pm \sqrt{2}, -1\right)$



17. The center is $(0, 0)$, $a = 3$, and $b = 2$, so an equation is $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. $c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(0, \pm\sqrt{5}\right)$.

18. The ellipse is centered at $(2, 1)$, with $a = 3$ and $b = 2$. An equation is $\dfrac{(x - 2)^2}{9} + \dfrac{(y - 1)^2}{4} = 1$.

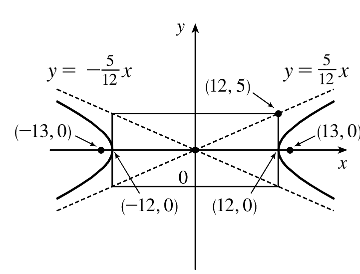
$c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(2 \pm \sqrt{5}, 1\right)$.

19. $\dfrac{x^2}{144} - \dfrac{y^2}{25} = 1 \quad\Rightarrow\quad a = 12, b = 5, c = \sqrt{144 + 25} = 13 \quad\Rightarrow$

center $(0, 0)$, vertices $(\pm 12, 0)$, foci $(\pm 13, 0)$,

asymptotes $y = \pm\frac{5}{12}x$.

Note: It is helpful to draw a $2a$-by-$2b$ rectangle whose center is the center of the hyperbola. The asymptotes are the extended diagonals of the rectangle.



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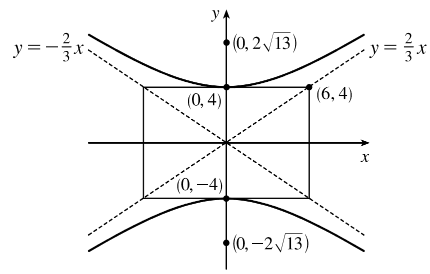
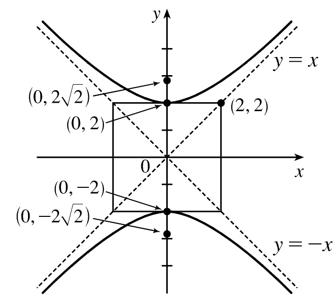
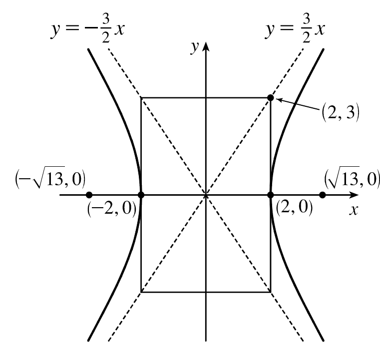
20. $\dfrac{y^2}{16} - \dfrac{x^2}{36} = 1 \quad \Rightarrow \quad a = 4, b = 6,$
$c = \sqrt{a^2 + b^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$. The center is $(0, 0)$, the vertices are $(0, \pm 4)$, the foci are $\left(0, \pm 2\sqrt{13}\right)$, and the asymptotes are the lines $y = \pm \frac{a}{b}x = \pm \frac{2}{3}x$.



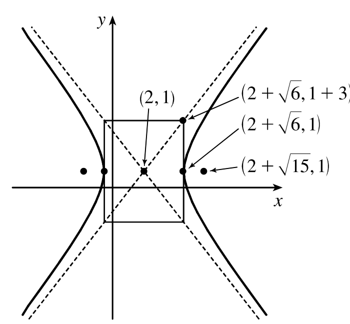
21. $y^2 - x^2 = 4 \quad \Leftrightarrow \quad \dfrac{y^2}{4} - \dfrac{x^2}{4} = 1 \quad \Rightarrow \quad a = \sqrt{4} = 2 = b,$
$c = \sqrt{4 + 4} = 2\sqrt{2} \quad \Rightarrow \quad$ center $(0, 0)$, vertices $(0, \pm 2)$, foci $\left(0, \pm 2\sqrt{2}\right)$, asymptotes $y = \pm x$



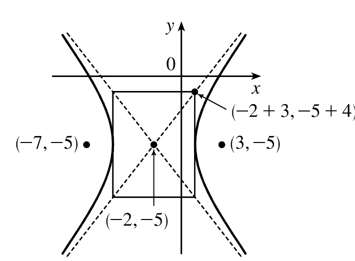
22. $9x^2 - 4y^2 = 36 \quad \Leftrightarrow \quad \dfrac{x^2}{4} - \dfrac{y^2}{9} = 1 \quad \Rightarrow \quad a = \sqrt{4} = 2,$
$b = \sqrt{9} = 3, c = \sqrt{4 + 9} = \sqrt{13} \quad \Rightarrow \quad$ center $(0, 0)$, vertices $(\pm 2, 0)$, foci $\left(\pm\sqrt{13}, 0\right)$, asymptotes $y = \pm \frac{3}{2}x$



23. $2y^2 - 4y - 3x^2 + 12x = -8 \quad \Leftrightarrow$
$2(y^2 - 2y + 1) - 3(x^2 - 4x + 4) = -8 + 2 - 12 \quad \Leftrightarrow$
$2(y - 1)^2 - 3(x - 2)^2 = -18 \quad \Leftrightarrow \quad \dfrac{(x - 2)^2}{6} - \dfrac{(y - 1)^2}{9} = 1$
$\Rightarrow \quad a = \sqrt{6}, b = 3, c = \sqrt{15} \quad \Rightarrow \quad$ center $(2, 1)$, vertices $\left(2 \pm \sqrt{6}, 1\right)$, foci $\left(2 \pm \sqrt{15}, 1\right)$, asymptotes $y - 1 = \pm \frac{3}{\sqrt{6}}(x - 2)$ or $y - 1 = \pm \dfrac{\sqrt{6}}{2}(x - 2)$



24. $16x^2 + 64x - 9y^2 - 90y = 305 \quad \Leftrightarrow$
$16(x^2 + 4x + 4) - 9(y^2 + 10y + 25) = 305 + 64 - 225 \quad \Leftrightarrow$
$16(x + 2)^2 - 9(y + 5)^2 = 144 \quad \Leftrightarrow \quad \dfrac{(x + 2)^2}{9} - \dfrac{(y + 5)^2}{16} = 1$
$\Rightarrow \quad a = 3, b = 4, c = 5 \quad \Rightarrow \quad$ center $(-2, -5)$, vertices $(-5, -5)$ and $(1, -5)$, foci $(-7, -5)$ and $(3, -5)$, asymptotes $y + 5 = \pm \frac{4}{3}(x + 2)$



25. $x^2 = y + 1 \quad \Leftrightarrow \quad x^2 = 1(y + 1)$. This is an equation of a *parabola* with $4p = 1$, so $p = \frac{1}{4}$. The vertex is $(0, -1)$ and the focus is $\left(0, -\frac{3}{4}\right)$.

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26. $x^2 = y^2 + 1 \quad\Leftrightarrow\quad x^2 - y^2 = 1$. This is an equation of a *hyperbola* with vertices $(\pm 1, 0)$. The foci are at $\left(\pm\sqrt{1+1}, 0\right) = \left(\pm\sqrt{2}, 0\right)$.

27. $x^2 = 4y - 2y^2 \quad\Leftrightarrow\quad x^2 + 2y^2 - 4y = 0 \quad\Leftrightarrow\quad x^2 + 2(y^2 - 2y + 1) = 2 \quad\Leftrightarrow\quad x^2 + 2(y-1)^2 = 2 \quad\Leftrightarrow$ $\dfrac{x^2}{2} + \dfrac{(y-1)^2}{1} = 1$. This is an equation of an *ellipse* with vertices at $\left(\pm\sqrt{2}, 1\right)$. The foci are at $\left(\pm\sqrt{2-1}, 1\right) = (\pm 1, 1)$.

28. $y^2 - 8y = 6x - 16 \quad\Leftrightarrow\quad y^2 - 8y + 16 = 6x \quad\Leftrightarrow\quad (y-4)^2 = 6x$. This is an equation of a *parabola* with $4p = 6$, so $p = \frac{3}{2}$. The vertex is $(0, 4)$ and the focus is $\left(\frac{3}{2}, 4\right)$.

29. $y^2 + 2y = 4x^2 + 3 \quad\Leftrightarrow\quad y^2 + 2y + 1 = 4x^2 + 4 \quad\Leftrightarrow\quad (y+1)^2 - 4x^2 = 4 \quad\Leftrightarrow\quad \dfrac{(y+1)^2}{4} - x^2 = 1$. This is an equation of a *hyperbola* with vertices $(0, -1 \pm 2) = (0, 1)$ and $(0, -3)$. The foci are at $\left(0, -1 \pm \sqrt{4+1}\right) = \left(0, -1 \pm \sqrt{5}\right)$.

30. $4x^2 + 4x + y^2 = 0 \quad\Leftrightarrow\quad 4\left(x^2 + x + \frac{1}{4}\right) + y^2 = 1 \quad\Leftrightarrow\quad 4\left(x + \frac{1}{2}\right)^2 + y^2 = 1 \quad\Leftrightarrow\quad \dfrac{\left(x+\frac{1}{2}\right)^2}{1/4} + y^2 = 1$. This is an equation of an *ellipse* with vertices $\left(-\frac{1}{2}, 0 \pm 1\right) = \left(-\frac{1}{2}, \pm 1\right)$. The foci are at $\left(-\frac{1}{2}, 0 \pm \sqrt{1 - \frac{1}{4}}\right) = \left(-\frac{1}{2}, \pm\sqrt{3}/2\right)$.

31. The parabola with vertex $(0, 0)$ and focus $(0, -2)$ opens downward and has $p = -2$, so its equation is $x^2 = 4py = -8y$.

32. The parabola with vertex $(1, 0)$ and directrix $x = -5$ opens to the right and has $p = 6$, so its equation is $y^2 = 4p(x-1) = 24(x-1)$.

33. The distance from the focus $(-4, 0)$ to the directrix $x = 2$ is $2 - (-4) = 6$, so the distance from the focus to the vertex is $\frac{1}{2}(6) = 3$ and the vertex is $(-1, 0)$. Since the focus is to the left of the vertex, $p = -3$. An equation is $y^2 = 4p(x+1) \quad\Rightarrow\quad y^2 = -12(x+1)$.

34. The distance from the focus $(3, 6)$ to the vertex $(3, 2)$ is $6 - 2 = 4$. Since the focus is above the vertex, $p = 4$. An equation is $(x-3)^2 = 4p(y-2) \quad\Rightarrow\quad (x-3)^2 = 16(y-2)$.

35. The parabola must have equation $y^2 = 4px$, so $(-4)^2 = 4p(1) \quad\Rightarrow\quad p = 4 \quad\Rightarrow\quad y^2 = 16x$.

36. Vertical axis $\quad\Rightarrow\quad (x-h)^2 = 4p(y-k)$. Substituting $(-2, 3)$ and $(0, 3)$ gives $(-2-h)^2 = 4p(3-k)$ and $(-h)^2 = 4p(3-k) \quad\Rightarrow\quad (-2-h)^2 = (-h)^2 \quad\Rightarrow\quad 4 + 4h + h^2 = h^2 \quad\Rightarrow\quad h = -1 \quad\Rightarrow\quad 1 = 4p(3-k)$. Substituting $(1, 9)$ gives $[1 - (-1)]^2 = 4p(9-k) \quad\Rightarrow\quad 4 = 4p(9-k)$. Solving for p from these equations gives $p = \dfrac{1}{4(3-k)} = \dfrac{1}{9-k} \quad\Rightarrow\quad 4(3-k) = 9-k \quad\Rightarrow\quad k = 1 \quad\Rightarrow\quad p = \frac{1}{8} \quad\Rightarrow\quad (x+1)^2 = \frac{1}{2}(y-1) \quad\Rightarrow$ $2x^2 + 4x - y + 3 = 0$.

37. The ellipse with foci $(\pm 2, 0)$ and vertices $(\pm 5, 0)$ has center $(0, 0)$ and a horizontal major axis, with $a = 5$ and $c = 2$, so $b = \sqrt{a^2 - c^2} = \sqrt{21}$. An equation is $\dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$.

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38. The ellipse with foci $(0, \pm 5)$ and vertices $(0, \pm 13)$ has center $(0, 0)$ and a vertical major axis, with $c = 5$ and $a = 13$, so $b = \sqrt{a^2 - c^2} = 12$. An equation is $\dfrac{x^2}{144} + \dfrac{y^2}{169} = 1$.

39. Since the vertices are $(0, 0)$ and $(0, 8)$, the ellipse has center $(0, 4)$ with a vertical axis and $a = 4$. The foci at $(0, 2)$ and $(0, 6)$ are 2 units from the center, so $c = 2$ and $b = \sqrt{a^2 - c^2} = \sqrt{4^2 - 2^2} = \sqrt{12}$. An equation is $\dfrac{(x-0)^2}{b^2} + \dfrac{(y-4)^2}{a^2} = 1 \quad \Rightarrow \quad \dfrac{x^2}{12} + \dfrac{(y-4)^2}{16} = 1$.

40. Since the foci are $(0, -1)$ and $(8, -1)$, the ellipse has center $(4, -1)$ with a horizontal axis and $c = 4$. The vertex $(9, -1)$ is 5 units from the center, so $a = 5$ and $b = \sqrt{a^2 - c^2} = \sqrt{5^2 - 4^2} = \sqrt{9}$. An equation is $\dfrac{(x-4)^2}{a^2} + \dfrac{(y+1)^2}{b^2} = 1 \quad \Rightarrow \quad \dfrac{(x-4)^2}{25} + \dfrac{(y+1)^2}{9} = 1$.

41. Center $(2, 2)$, $c = 2$, $a = 3 \quad \Rightarrow \quad b = \sqrt{5} \quad \Rightarrow \quad \frac{1}{9}(x-2)^2 + \frac{1}{5}(y-2)^2 = 1$

42. Center $(0, 0)$, $c = 2$, major axis horizontal $\quad \Rightarrow \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and $b^2 = a^2 - c^2 = a^2 - 4$. Since the ellipse passes through $(2, 1)$, we have $2a = |PF_1| + |PF_2| = \sqrt{17} + 1 \quad \Rightarrow \quad a^2 = \frac{9+\sqrt{17}}{2}$ and $b^2 = \frac{1+\sqrt{17}}{2}$, so the ellipse has equation $\dfrac{2x^2}{9+\sqrt{17}} + \dfrac{2y^2}{1+\sqrt{17}} = 1$.

43. Center $(0, 0)$, vertical axis, $c = 3$, $a = 1 \quad \Rightarrow \quad b = \sqrt{8} = 2\sqrt{2} \quad \Rightarrow \quad y^2 - \frac{1}{8}x^2 = 1$

44. Center $(0, 0)$, horizontal axis, $c = 6$, $a = 4 \quad \Rightarrow \quad b = 2\sqrt{5} \quad \Rightarrow \quad \frac{1}{16}x^2 - \frac{1}{20}y^2 = 1$

45. Center $(4, 3)$, horizontal axis, $c = 3$, $a = 2 \quad \Rightarrow \quad b = \sqrt{5} \quad \Rightarrow \quad \frac{1}{4}(x-4)^2 - \frac{1}{5}(y-3)^2 = 1$

46. Center $(2, 3)$, vertical axis, $c = 5$, $a = 3 \quad \Rightarrow \quad b = 4 \quad \Rightarrow \quad \frac{1}{9}(y-3)^2 - \frac{1}{16}(x-2)^2 = 1$

47. Center $(0, 0)$, horizontal axis, $a = 3$, $\frac{b}{a} = 2 \quad \Rightarrow \quad b = 6 \quad \Rightarrow \quad \frac{1}{9}x^2 - \frac{1}{36}y^2 = 1$

48. Center $(4, 2)$, horizontal axis, asymptotes $y - 2 = \pm(x - 4) \quad \Rightarrow \quad c = 2$, $b/a = 1 \quad \Rightarrow \quad a = b \quad \Rightarrow$ $c^2 = 4 = a^2 + b^2 = 2a^2 \quad \Rightarrow \quad a^2 = 2 \quad \Rightarrow \quad \frac{1}{2}(x-4)^2 - \frac{1}{2}(y-2)^2 = 1$

49. In Figure 8, we see that the point on the ellipse closest to a focus is the closer vertex (which is a distance $a - c$ from it) while the farthest point is the other vertex (at a distance of $a + c$). So for this lunar orbit, $(a - c) + (a + c) = 2a = (1728 + 110) + (1728 + 314)$, or $a = 1940$; and $(a + c) - (a - c) = 2c = 314 - 110$, or $c = 102$. Thus, $b^2 = a^2 - c^2 = 3{,}753{,}196$, and the equation is $\dfrac{x^2}{3{,}763{,}600} + \dfrac{y^2}{3{,}753{,}196} = 1$.

50. (a) Choose V to be the origin, with x-axis through V and F. Then F is $(p, 0)$, A is $(p, 5)$, so substituting A into the equation $y^2 = 4px$ gives $25 = 4p^2$ so $p = \frac{5}{2}$ and $y^2 = 10x$.

(b) $x = 11 \quad \Rightarrow \quad y = \sqrt{110} \quad \Rightarrow \quad |CD| = 2\sqrt{110}$

51. (a) Set up the coordinate system so that A is $(-200, 0)$ and B is $(200, 0)$.
$|PA| - |PB| = (1200)(980) = 1{,}176{,}000 \text{ ft} = \frac{2450}{11} \text{ mi} = 2a \quad \Rightarrow \quad a = \frac{1225}{11}$, and $c = 200$ so $b^2 = c^2 - a^2 = \frac{3{,}339{,}375}{121} \quad \Rightarrow \quad \dfrac{121x^2}{1{,}500{,}625} - \dfrac{121y^2}{3{,}339{,}375} = 1$.

(b) Due north of $B \quad \Rightarrow \quad x = 200 \quad \Rightarrow \quad \dfrac{(121)(200)^2}{1{,}500{,}625} - \dfrac{121y^2}{3{,}339{,}375} = 1 \quad \Rightarrow \quad y = \dfrac{133{,}575}{539} \approx 248$ mi

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52. $|PF_1| - |PF_2| = \pm 2a \quad \Leftrightarrow \quad \sqrt{(x+c)^2+y^2} - \sqrt{(x-c)^2+y^2} = \pm 2a \quad \Leftrightarrow$

$\sqrt{(x+c)^2+y^2} = \sqrt{(x-c)^2+y^2} \pm 2a \quad \Leftrightarrow \quad (x+c)^2+y^2 = (x-c)^2+y^2+4a^2 \pm 4a\sqrt{(x-c)^2+y^2}$

$\Leftrightarrow \quad 4cx - 4a^2 = \pm 4a\sqrt{(x-c)^2+y^2} \quad \Leftrightarrow \quad c^2x^2 - 2a^2cx + a^4 = a^2\left(x^2 - 2cx + c^2 + y^2\right) \quad \Leftrightarrow$

$\left(c^2 - a^2\right)x^2 - a^2y^2 = a^2\left(c^2 - a^2\right) \quad \Leftrightarrow \quad b^2x^2 - a^2y^2 = a^2b^2$ (where $b^2 = c^2 - a^2$) $\quad \Leftrightarrow \quad \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

53. The function whose graph is the upper branch of this hyperbola is concave upward. The function is $y = f(x) = a\sqrt{1 + \dfrac{x^2}{b^2}} = \dfrac{a}{b}\sqrt{b^2+x^2}$, so $y' = \dfrac{a}{b}x\left(b^2+x^2\right)^{-1/2}$ and

$y'' = \frac{a}{b}\left[\left(b^2+x^2\right)^{-1/2} - x^2\left(b^2+x^2\right)^{-3/2}\right] = ab\left(b^2+x^2\right)^{-3/2} > 0$ for all x, and so f is concave upward.

54. We can follow exactly the same sequence of steps as in the derivation of Formula 4, except we use the points $(1,1)$ and $(-1,-1)$ in the distance formula (first equation of that derivation) so $\sqrt{(x-1)^2+(y-1)^2} + \sqrt{(x+1)^2+(y+1)^2} = 4$ will lead (after moving the second term to the right, squaring, and simplifying) to $2\sqrt{(x+1)^2+(y+1)^2} = x + y + 4$, which, after squaring and simplifying again, leads to $3x^2 - 2xy + 3y^2 = 8$.

55. (a) If $k > 16$, then $k - 16 > 0$, and $\dfrac{x^2}{k} + \dfrac{y^2}{k-16} = 1$ is an *ellipse* since it is the sum of two squares on the left side.

(b) If $0 < k < 16$, then $k - 16 < 0$, and $\dfrac{x^2}{k} + \dfrac{y^2}{k-16} = 1$ is a *hyperbola* since it is the difference of two squares on the left side.

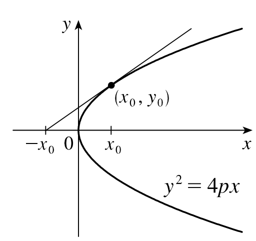
(c) If $k < 0$, then $k - 16 < 0$, and there is *no curve* since the left side is the sum of two negative terms, which cannot equal 1.

(d) In case (a), $a^2 = k$, $b^2 = k - 16$, and $c^2 = a^2 - b^2 = 16$, so the foci are at $(\pm 4, 0)$. In case (b), $k - 16 < 0$, so $a^2 = k$, $b^2 = 16 - k$, and $c^2 = a^2 + b^2 = 16$, and so again the foci are at $(\pm 4, 0)$.

56. (a) $y^2 = 4px \quad \Rightarrow \quad 2yy' = 4p \quad \Rightarrow \quad y' = \dfrac{2p}{y}$, so the tangent line is

$y - y_0 = \dfrac{2p}{y_0}(x - x_0) \quad \Rightarrow \quad yy_0 - y_0^2 = 2p(x - x_0) \quad \Leftrightarrow$

$yy_0 - 4px_0 = 2px - 2px_0 \quad \Rightarrow \quad yy_0 = 2p(x + x_0)$.



(b) The x-intercept is $-x_0$.

57. Use the parametrization $x = 2\cos t$, $y = \sin t$, $0 \le t \le 2\pi$ to get

$$L = 4\int_0^{\pi/2}\sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt = 4\int_0^{\pi/2}\sqrt{4\sin^2 t + \cos^2 t}\,dt = 4\int_0^{\pi/2}\sqrt{3\sin^2 t + 1}\,dt$$

Using Simpson's Rule with $n = 10$, $\Delta t = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$, and $f(t) = \sqrt{3\sin^2 t + 1}$, we get

$$L \approx \tfrac{4}{3}\left(\tfrac{\pi}{20}\right)\left[f(0) + 4f\left(\tfrac{\pi}{20}\right) + 2f\left(\tfrac{2\pi}{20}\right) + \cdots + 2f\left(\tfrac{8\pi}{20}\right) + 4f\left(\tfrac{9\pi}{20}\right) + f\left(\tfrac{\pi}{2}\right)\right] \approx 9.69$$

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58. The length of the major axis is $2a$, so $a = \frac{1}{2}(1.18 \times 10^{10}) = 5.9 \times 10^9$. The length of the minor axis is $2b$, so $b = \frac{1}{2}(1.14 \times 10^{10}) = 5.7 \times 10^9$. An equation of the ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, or converting into parametric equations, $x = a\cos\theta$ and $y = b\sin\theta$. So

$$L = 4\int_0^{\pi/2} \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\, d\theta = 4\int_0^{\pi/2} \sqrt{a^2 \sin^2\theta + b^2\cos^2\theta}\, d\theta$$

Using Simpson's Rule with $n = 10$, $\Delta\theta = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$, and $f(\theta) = \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, we get

$$\begin{aligned} L &\approx 4 \cdot S_{10} \\ &= 4 \cdot \tfrac{\pi}{20 \cdot 3}\left[f(0) + 4f\left(\tfrac{\pi}{20}\right) + 2f\left(\tfrac{2\pi}{20}\right) + \cdots + 2f\left(\tfrac{8\pi}{20}\right) + 4f\left(\tfrac{9\pi}{20}\right) + f\left(\tfrac{\pi}{2}\right)\right] \\ &\approx 3.64 \times 10^{10} \text{ km} \end{aligned}$$

59. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \;\Rightarrow\; \dfrac{2x}{a^2} + \dfrac{2yy'}{b^2} = 0 \;\Rightarrow\; y' = -\dfrac{b^2x}{a^2y}$ $(y \neq 0)$. Thus, the slope of the tangent line at P is $-\dfrac{b^2x_1}{a^2y_1}$. The slope of F_1P is $\dfrac{y_1}{x_1+c}$ and of F_2P is $\dfrac{y_1}{x_1-c}$. By the formula from Problems Plus, we have

$$\tan\alpha = \frac{\dfrac{y_1}{x_1+c} + \dfrac{b^2x_1}{a^2y_1}}{1 - \dfrac{b^2x_1y_1}{a^2y_1(x_1+c)}} = \frac{a^2y_1^2 + b^2x_1(x_1+c)}{a^2y_1(x_1+c) - b^2x_1y_1} = \frac{a^2b^2 + b^2cx_1}{c^2x_1y_1 + a^2cy_1} \quad \begin{bmatrix}\text{using } b^2x_1^2 + a^2y_1^2 = a^2b^2 \\ \text{and } a^2 - b^2 = c^2\end{bmatrix}$$

$$= \frac{b^2(cx_1 + a^2)}{cy_1(cx_1 + a^2)} = \frac{b^2}{cy_1}$$

and

$$\tan\beta = \frac{-\dfrac{y_1}{x_1-c} - \dfrac{b^2x_1}{a^2y_1}}{1 - \dfrac{b^2x_1y_1}{a^2y_1(x_1-c)}} = \frac{-a^2y_1^2 - b^2x_1(x_1-c)}{a^2y_1(x_1-c) - b^2x_1y_1} = \frac{-a^2b^2 + b^2cx_1}{c^2x_1y_1 - a^2cy_1} = \frac{b^2(cx_1 - a^2)}{cy_1(cx_1 - a^2)} = \frac{b^2}{cy_1}$$

So $\alpha = \beta$.

60. The slopes of the line segments F_1P and F_2P are $\dfrac{y_1}{x_1+c}$ and $\dfrac{y_1}{x_1-c}$, where P is (x_1, y_1). Differentiating implicitly, $\dfrac{2x}{a^2} - \dfrac{2yy'}{b^2} = 0 \;\Rightarrow\; y' = \dfrac{b^2x}{a^2y} \;\Rightarrow\;$ the slope of the tangent at P is $\dfrac{b^2x_1}{a^2y_1}$, so by the formula from Problems Plus,

$$\tan\alpha = \frac{\dfrac{b^2x_1}{a^2y_1} - \dfrac{y_1}{x_1+c}}{1 + \dfrac{b^2x_1y_1}{a^2y_1(x_1+c)}} = \frac{b^2x_1(x_1+c) - a^2y_1^2}{a^2y_1(x_1+c) + b^2x_1y_1}$$

$$= \frac{b^2(cx_1 + a^2)}{cy_1(cx_1 + a^2)} \quad \begin{bmatrix}\text{using } x_1^2/a^2 - y_1^2/b^2 = 1 \\ \text{and } a^2 + b^2 = c^2\end{bmatrix} \quad = \frac{b^2}{cy_1}$$

and

$$\tan\beta = \frac{-\dfrac{b^2x_1}{a^2y_1} + \dfrac{y_1}{x_1-c}}{1 + \dfrac{b^2x_1y_1}{a^2y_1(x_1-c)}} = \frac{-b^2x_1(x_1-c) + a^2y_1^2}{a^2y_1(x_1-c) + b^2x_1y_1} = \frac{b^2(cx_1 - a^2)}{cy_1(cx_1 - a^2)} = \frac{b^2}{cy_1}$$

So $\alpha = \beta$.

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