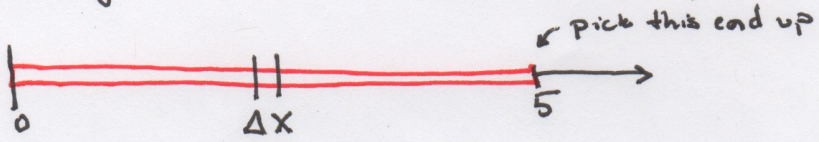
Q. A chain lying on the ground is 5m long and weighs 100 kg. How much work is required to raise one end of the chain to a height of 7m?

A. Imagine the chain coiled up, but we lay it out straight to set a coordinate system:



The force exherted by each segment is

$$\boxed{dF = \left(\frac{100\ kg}{5m}\right)\underbrace{dx}_{m}\ (9.8\ m/s^2)}$$

The right end gets lifted 7m.
$\uparrow_{(x=5)}$

The left end gets lifted 2m
$\uparrow_{(x=0)}$

So the segment @ point x gets lifted

$$\boxed{x+2 \quad m.}$$

So $\boxed{dW = \left(\frac{100}{5}\right)(9.8)(x+2)\,dx}$,

and thus

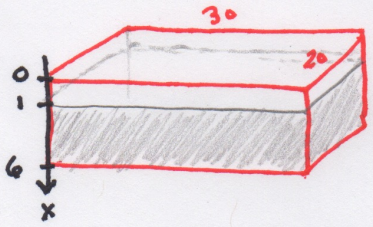
$$W = \int_0^5 \left(\frac{100}{5}\right)(9.8)(x+2)\,dx$$

$$= 20(9.8)\left(x^2/2 + 2x\right)\Big|_0^5$$

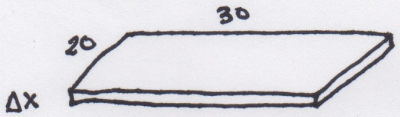
$$= \boxed{20(9.8)\left(\frac{25}{2} + 10\right)\ \frac{kg \cdot m^2}{s^2}}$$

Q. A rectangular swimming pool has sides of length 20 ft and 30 ft, and a constant height of 6 ft. It is filled to a depth of 5 ft with water (which weighs 62.5 lb/ft^3). How much work is required to pump all of the water out over the side (to the top)?

A. Let x be the depth of the water



For each fixed x, the water at that height all has the same distance to travel. So slice the water horizontally.



$dV = 20 \cdot 30\, dx$

The force exerted by that slice is

$$dF = (62.5)(20)(30)\, dx.$$

The slice at depth x has x ft to travel.

So $dW = (62.5)(20)(30)\, x\, dx$

So

$$W = \int_1^6 (62.5)(20)(30)\, x\, dx$$

$$= (62.5)(20)(30)\, \frac{x^2}{2} \Big|_1^6$$

$$= (62.5)(20)(30)\left(\frac{36}{2} - \frac{1}{2}\right) \text{ ft} \cdot \text{lb}$$