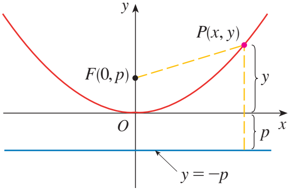
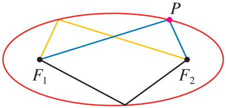
Review of conic sections

Conic sections are graphs of the form

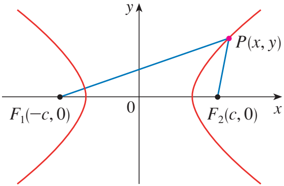
parabolas



ellipses



hyperbolas



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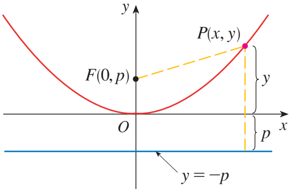

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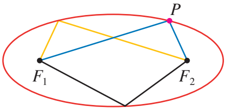

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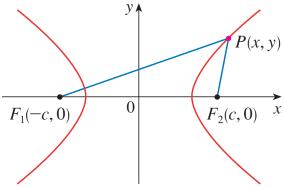
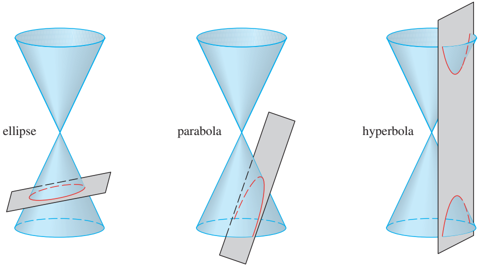


They're called conic sections because they can be found by taking a doubled cone and slicing it with a plane:



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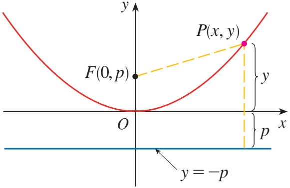

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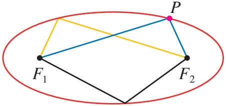

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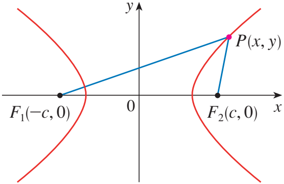
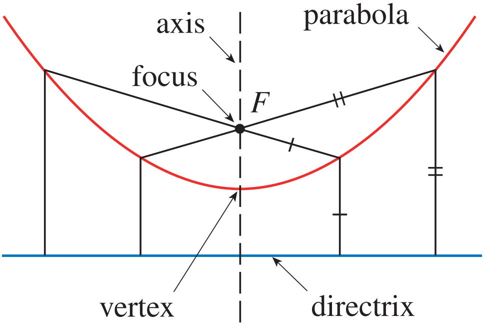


All conic sections satisfy an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

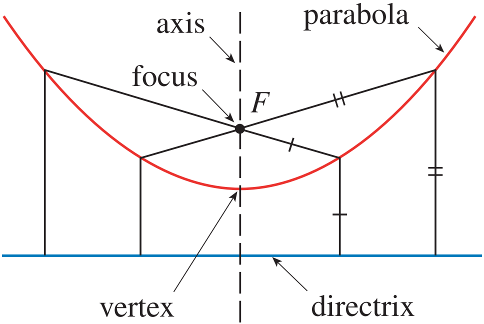
for some constants A, B, C, D, E, F.

Parabolas



Parabolas have a focus F (a point) and a directrix (a line), and are defined as all the points that are equidistant from the focus and the directrix.

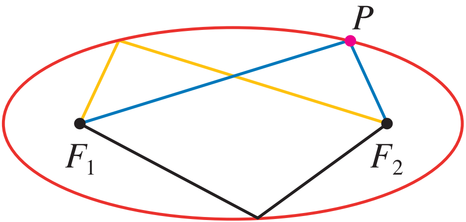
Parabolas



Parabolas have a focus F (a point) and a directrix (a line), and are defined as all the points that are equidistant from the focus and the directrix.

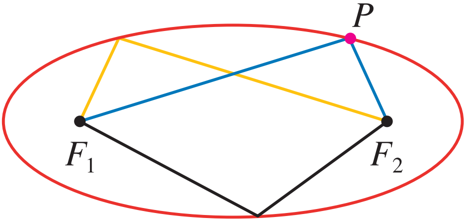
They also have a vertex (the point on the curve closest to the directrix) and an axis (the line perpendicular to the directrix through the vertex and the focus).

Ellipses



Ellipses have two foci F_1 and F_2 (both points), and are defined as all the points (x, y) such that the distance from F_1 to (x, y) plus the distance from (x, y) to F_2 is fixed.

Ellipses



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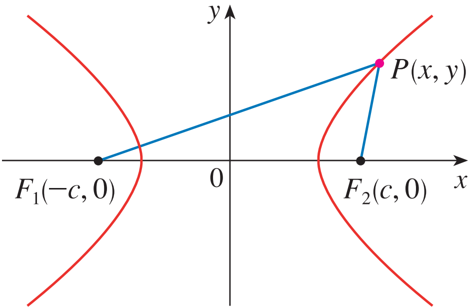
They also have a major axis (the line through the foci) and vertices (the points on the curve intersecting the major axis).

Hyperbolas



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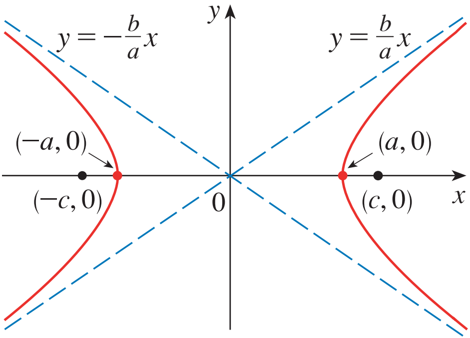

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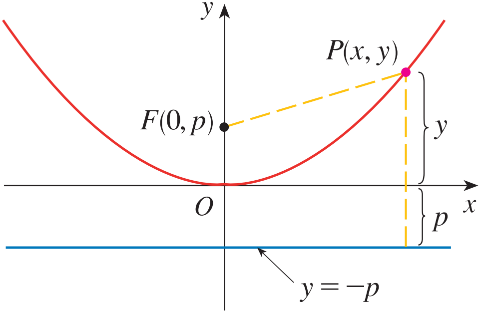
They also have vertices (one point on each piece which are closest to each other) and asymptotes (lines which the curves approach at infinity).

Calculating equations for conic sections: Parabolas



For a parabola with vertex at the origin and directrix $y = -p$ parallel to the x-axis, the definition "all the points $P(x, y)$ that are equidistant from the focus and the directrix" says first that the focus had better be at $(0, p)$ (since the origin is on the curve)

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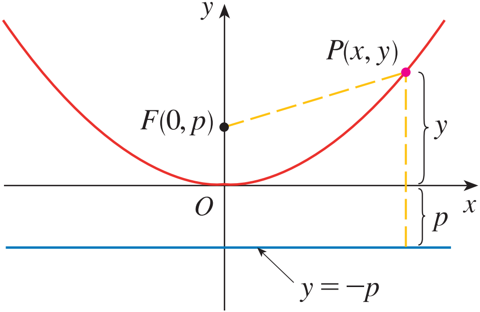

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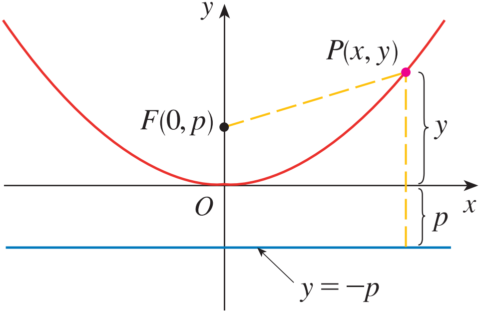

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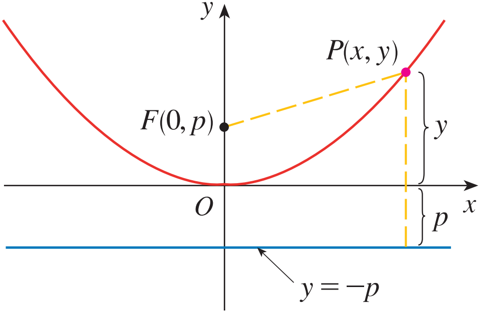

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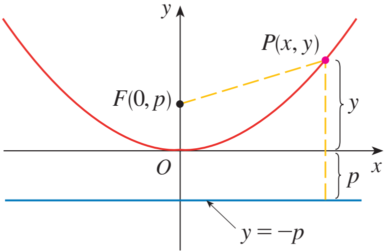

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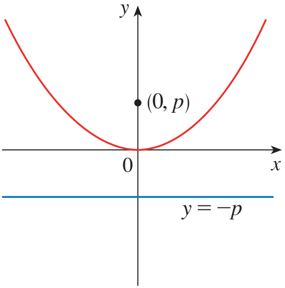
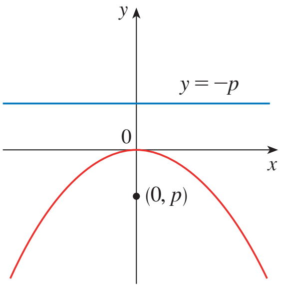
Solving for x^2, this gives $\boxed{x^2 = 4py}$. Similarly, if the parabola had vertex at the origin and directrix $x = -p$, then the focus is at $(p, 0)$ and the equation for the parabola is $\boxed{y^2 = 4px}$.



$x^2 = 4py, p > 0$



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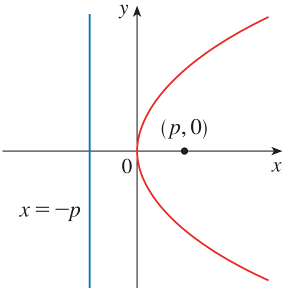

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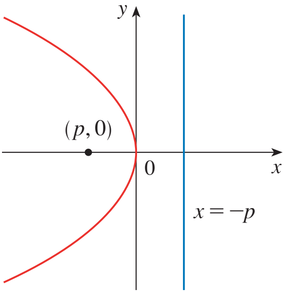

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$$-x^2-4x$$

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3. What conic section is the curve $10y - x^2 - 4x - 14 = 0$?
Ans. Try to put in a form we recognize! Deal with the x and y stuff separately and complete whatever squares appear.

$$\begin{aligned} -x^2 - 4x &= -(x^2+4x) = -(x^2+4x+4-4) \\ &= -((x+2)^2-4) = \boxed{-(x+2)^2+4}; \end{aligned}$$

So

$$\begin{aligned} 0 &= 10y + \boxed{-x^2-4x} - 14 \\ &= 10y + \boxed{-(x+2)^2+4} - 14 = 10(y-1) - (x+2)^2. \end{aligned}$$

Same as part 2!

Doing parabola problems

Look for x^2 or y^2, but not both!

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Get into the proper form!

$$(\hat{x})^2 = 4p\hat{y} \text{ or } (\hat{y})^2 = 4p\hat{x}.$$

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$$(\hat{x})^2 = 4p\hat{y} \text{ or } (\hat{y})^2 = 4p\hat{x}.$$

If you're given something like $10y - x^2 - 4x - 14 = 0$, separate the x stuff and the y stuff and complete squares, and deal with shifting coordinates if necessary.

1–8 ■ Find the vertex, focus, and directrix of the parabola and sketch its graph.

1. $x = 2y^2$

2. $4y + x^2 = 0$

3. $4x^2 = -y$

4. $y^2 = 12x$

5. $(x + 2)^2 = 8(y - 3)$

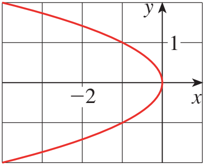
6. $x - 1 = (y + 5)^2$

7. $y^2 + 2y + 12x + 25 = 0$

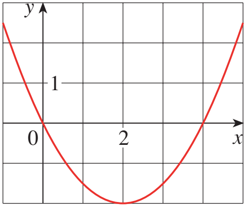
8. $y + 12x - 2x^2 = 16$

9–10 ■ Find an equation of the parabola. Then find the focus and directrix.

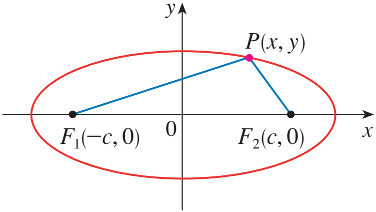
9.



10.



Calculating equations for conic sections: Ellipses



For an ellipse with the foci on the x-axis at the points $F_1 = (-c, 0)$ and $F_2 = (c, 0)$ and the sum of the distances from a point on the ellipse to the foci be $2a > 0$. Then for any point $P = P(x, y)$ on the curve

$$2a = |PF_1| + |PF_2|$$

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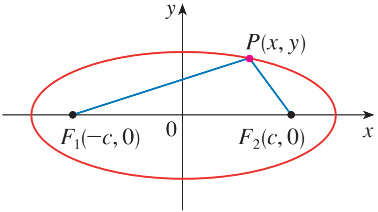

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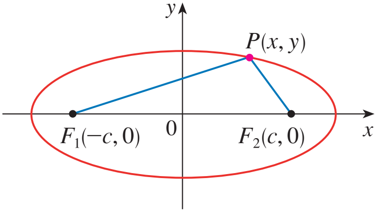

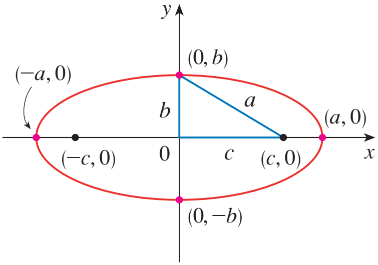

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Since $a > c$, let b be defined by $b^2 = a^2 - c^2$.

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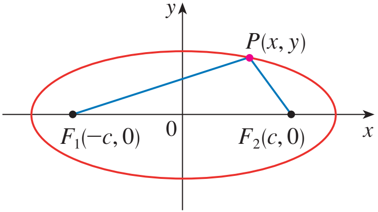

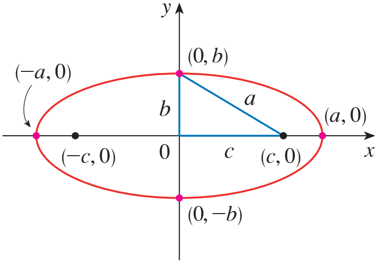

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Since $a > c$, let b be defined by $b^2 = a^2 - c^2$. Manipulating the equation above gives

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1.$$

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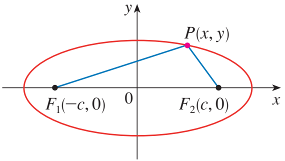

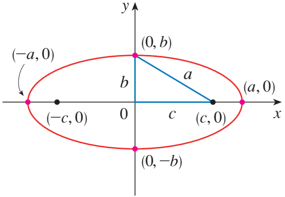

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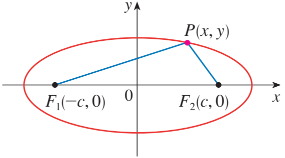
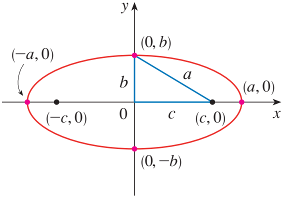
Calculating equations for conic sections: Ellipses





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The major axis is the x-axis and the vertices are $(\pm a, 0)$.
Similarly, if the foci are the points $F_1 = (0, -c)$ and $F_2 = (0, c)$ and the sum of the distances from a point on the ellipse to the foci be $2a > 0$,

$$\left(\frac{x}{b}\right)^2 + \left(\frac{y}{a}\right)^2 = 1, \text{ where } b^2 = a^2 - c^2.$$

The major axis is the y-axis and the vertices are $(0, \pm a)$.

Doing ellipse problems

Look for both x^2 and y^2, with the same sign.

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$$9x^2 + 16y^2 = 144,$$

divide both sides by 144 first and factor into squares:

$$1 = \frac{9}{144}x^2 + \frac{16}{144}y^2$$

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If you're given something like

$$y^2 + 2x^2 - 2y + 8x = 0$$

separate the x stuff and the y stuff and complete squares, and deal with shifting coordinates if necessary.

11–16 ■ Find the vertices and foci of the ellipse and sketch its graph.

11. $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$

12. $\dfrac{x^2}{64} + \dfrac{y^2}{100} = 1$

13. $4x^2 + y^2 = 16$

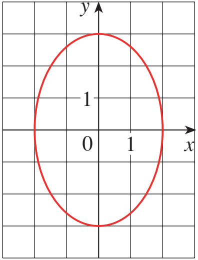
14. $4x^2 + 25y^2 = 25$

15. $9x^2 - 18x + 4y^2 = 27$

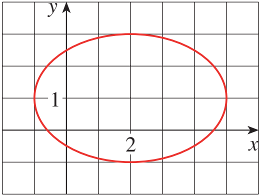
16. $x^2 + 2y^2 - 6x + 4y + 7 = 0$

17–18 ■ Find an equation of the ellipse. Then find its foci.

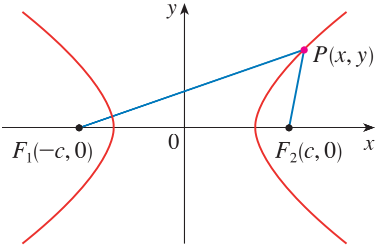
17.



18.



Calculating equations for conic sections: Hyperbolas



For a hyperbola with the foci on the x-axis at the points $F_1 = (-c, 0)$ and $F_2 = (c, 0)$ and the difference of the distances from a point on the ellipse to the foci be $\pm 2a$ $(a > 0)$.

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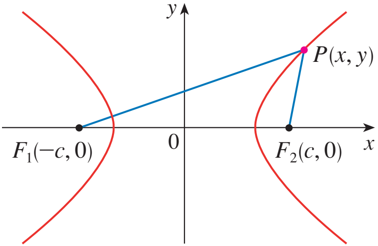

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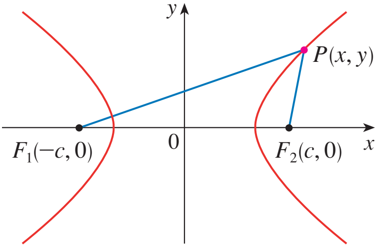

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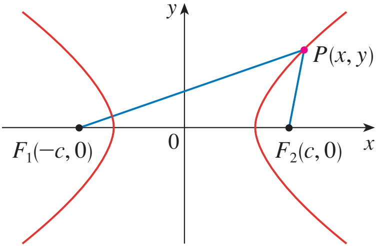

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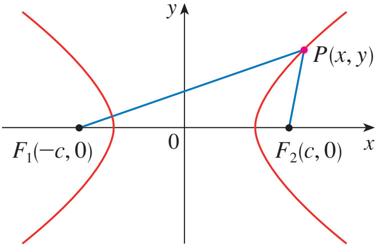

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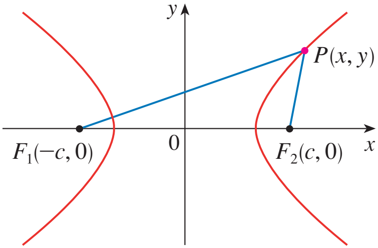

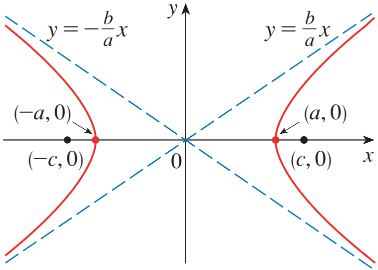

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Note that the x-intercepts are $\pm a$ (set $y = 0$ and solve). There is no y-intercept ($x = 0$ has no solutions).

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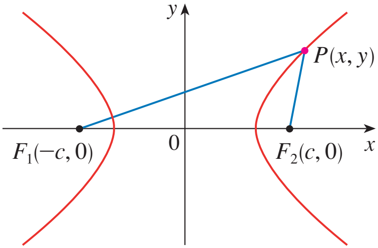

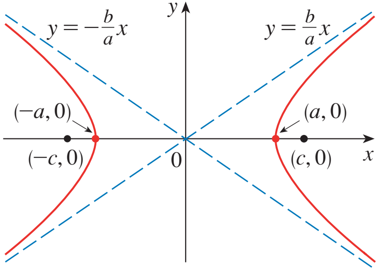

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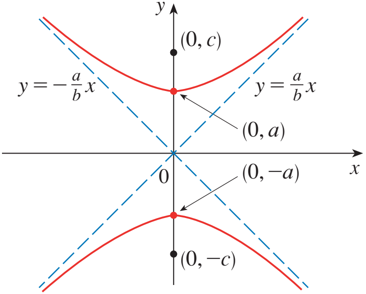
Calculating equations for conic sections: Hyperbolas



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The y-intercepts are $\pm a$ (set $x = 0$ and solve). There is no x-intercept ($y = 0$ has no solutions). And as $x \to \pm\infty$, $y/x \to \pm a/b$. So the vertices are $(0, \pm a)$ and the asymptotes are $y = \pm(a/b)x$. (Switch all x's for y's.)

Doing hyperbola problems

Look for both x^2 and y^2, with different signs.

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Look for both x^2 and y^2, with different signs.

Get into the proper form!

$$\left(\frac{\hat{x}}{a}\right)^2 - \left(\frac{\hat{y}}{b}\right)^2 = 1 \text{ or } \left(\frac{\hat{y}}{a}\right)^2 - \left(\frac{\hat{x}}{b}\right)^2 = 1$$

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For example, if you're given

$$9x^2 - 16y^2 = 144,$$

divide both sides by 144 first and factor into squares:

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If you're given something like

$$y^2 - 2x^2 - 2y - 8x = 0$$

separate the x stuff and the y stuff and complete squares, and deal with shifting coordinates if necessary.

19–20 ■ Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph.

19. $\dfrac{x^2}{144} - \dfrac{y^2}{25} = 1$

20. $\dfrac{y^2}{16} - \dfrac{x^2}{36} = 1$

21. $y^2 - x^2 = 4$

22. $9x^2 - 4y^2 = 36$

23. $2y^2 - 3x^2 - 4y + 12x + 8 = 0$

24. $16x^2 - 9y^2 + 64x - 90y = 305$

25–30 ■ Identify the type of conic section whose equation is given and find the vertices and foci.

25. $x^2 = y + 1$

26. $x^2 = y^2 + 1$

27. $x^2 = 4y - 2y^2$

28. $y^2 - 8y = 6x - 16$

29. $y^2 + 2y = 4x^2 + 3$

30. $4x^2 + 4x + y^2 = 0$