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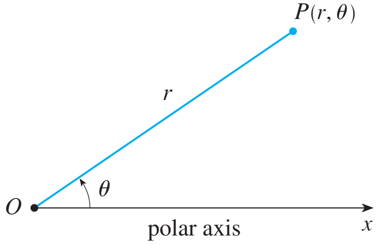

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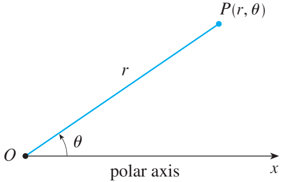

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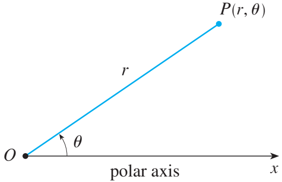
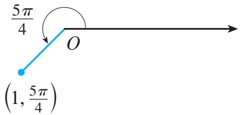


For example,

$$(r, \theta) = (1, 5\pi/4)$$



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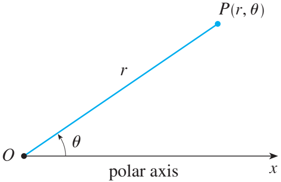

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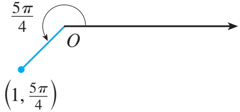

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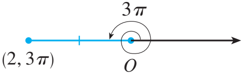

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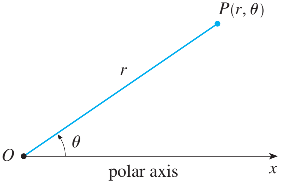

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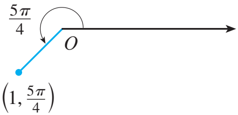

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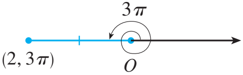

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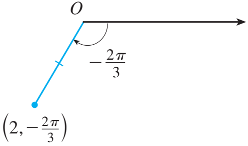

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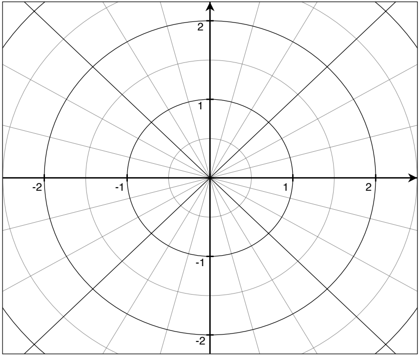

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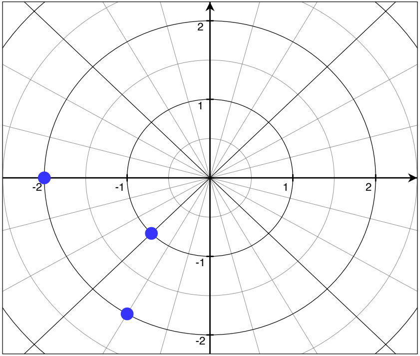

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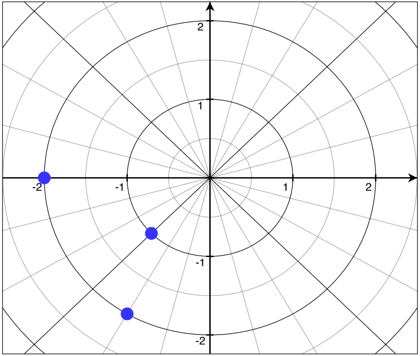

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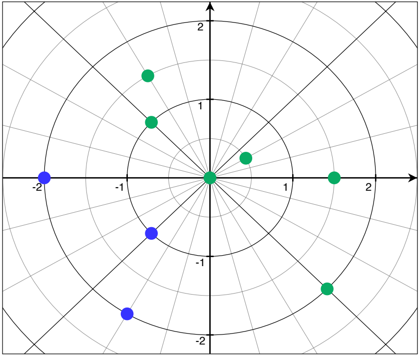

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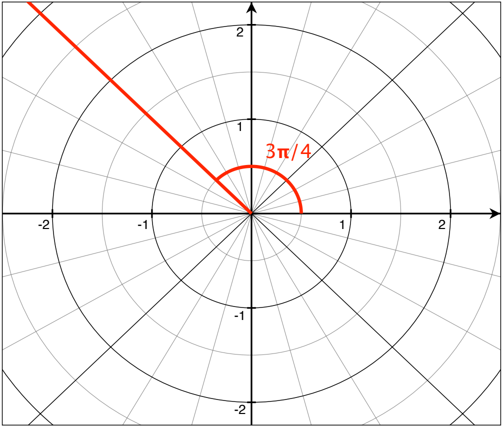

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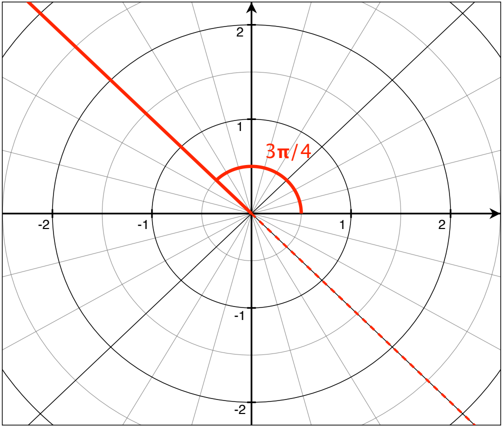

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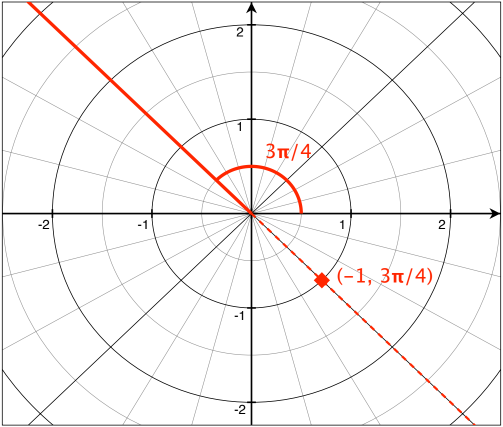

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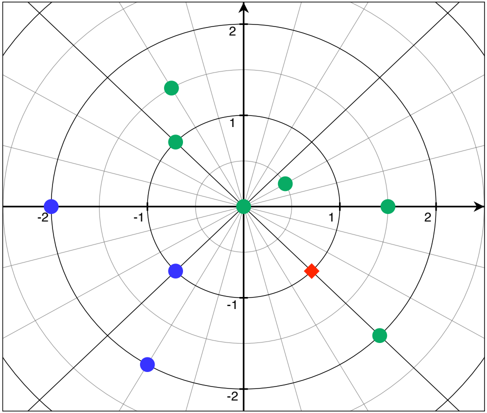

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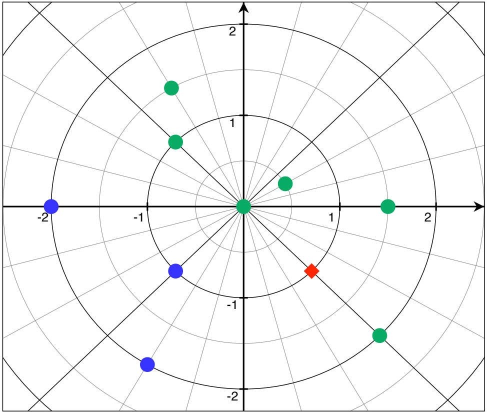

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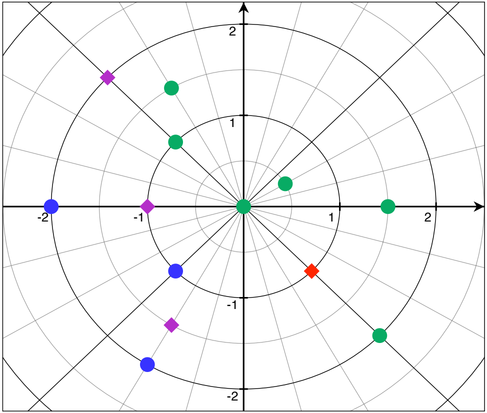

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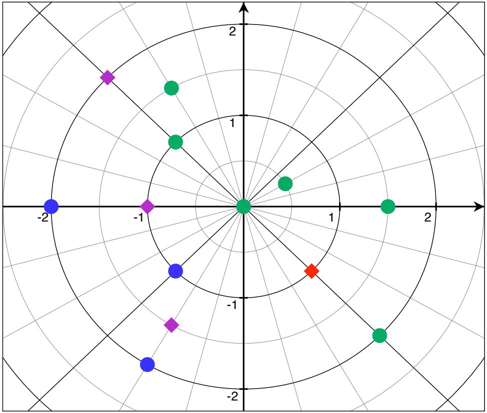

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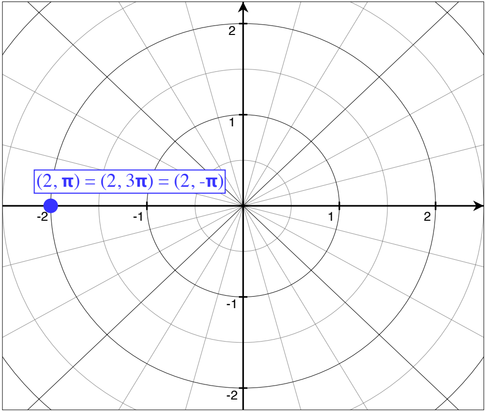

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$$(r, \theta) = (r, \theta + k \cdot 2\pi) \quad \text{for } k = 0, \pm 1, \pm 2, \dots;$$

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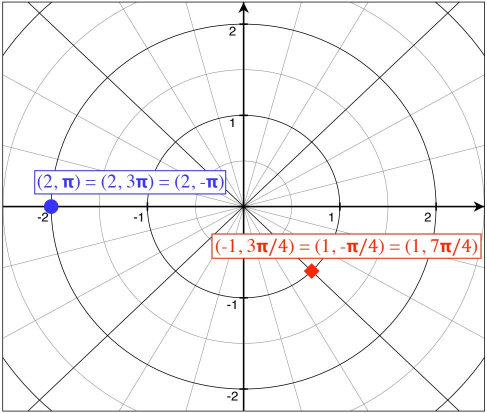

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Just like we used to graph $y = f(x)$ or $x = g(y)$, now we graph things like $r = f(\theta)$ or $\theta = g(r)$.

Constant graphs:

$$r = \text{constant}$$

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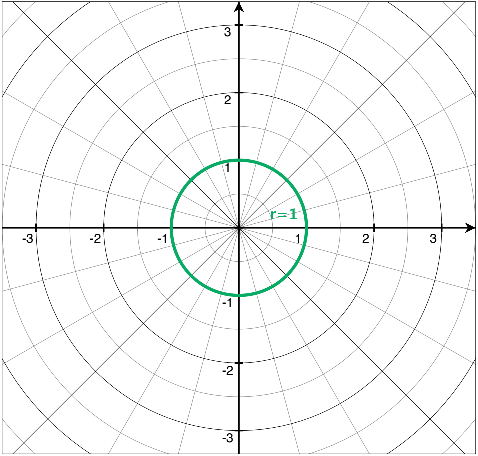

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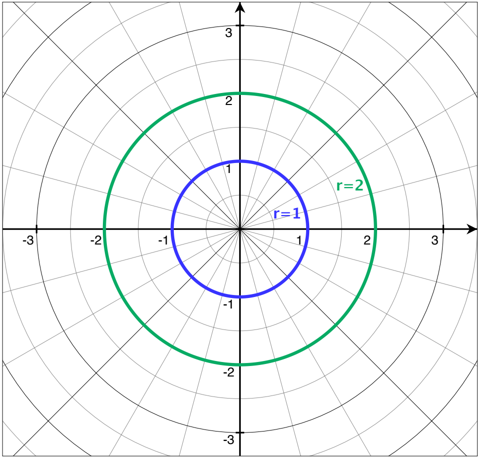

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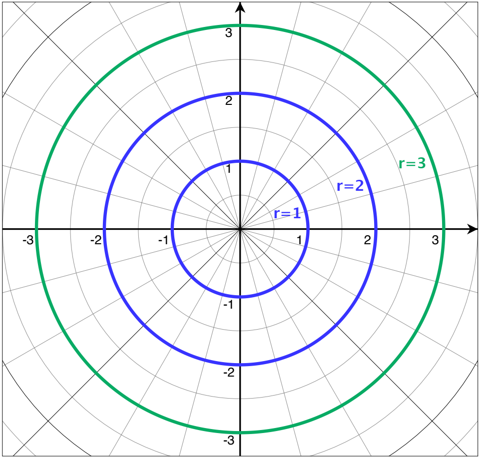

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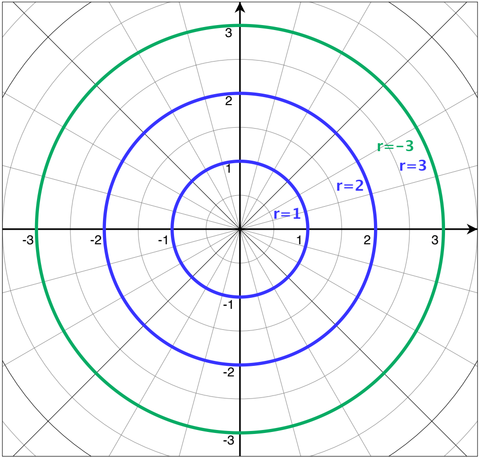

Note:
$r = c$
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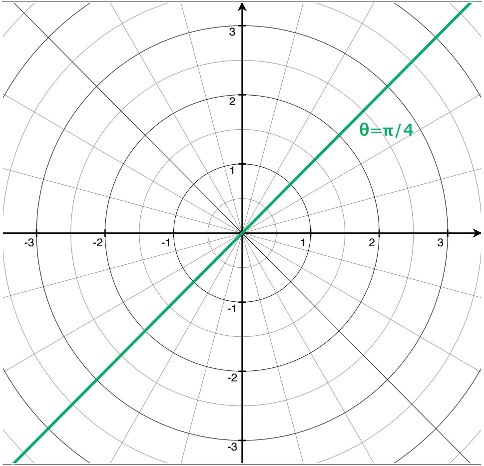

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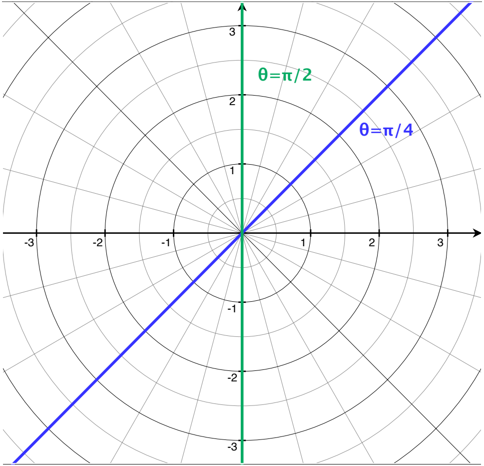

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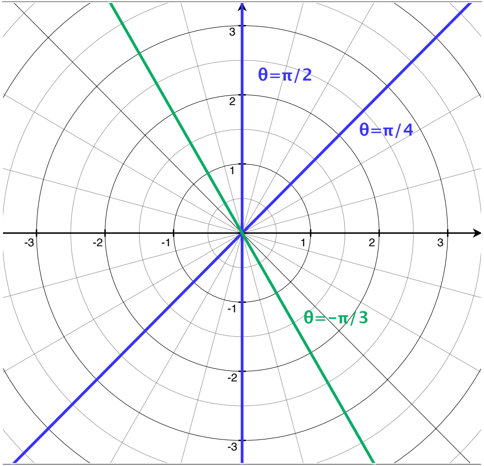

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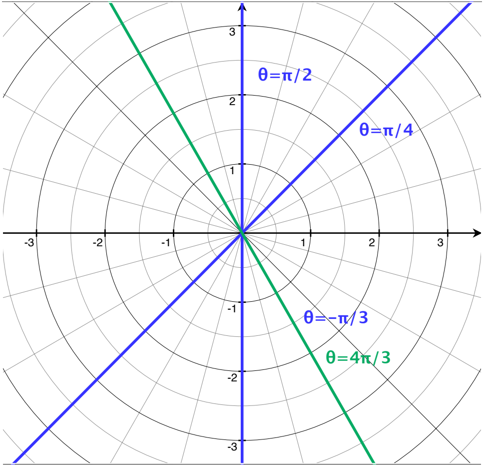

Note:
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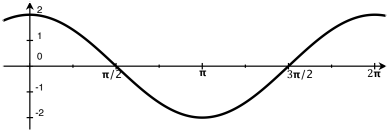
First, on a Cartesian (θ, r) plot, this function looks like



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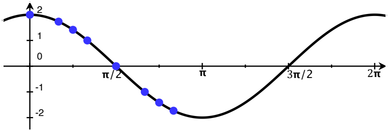
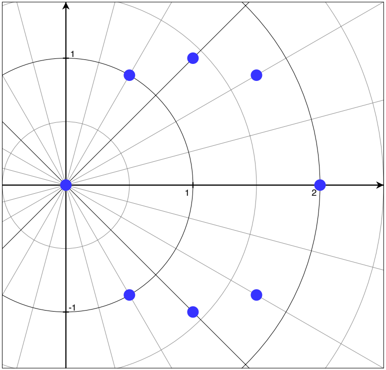


(1) Plot points



θ	r
0	2
$\pi/6$	$\sqrt{3}$
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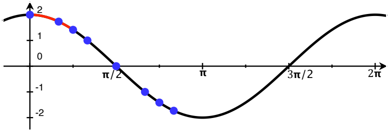

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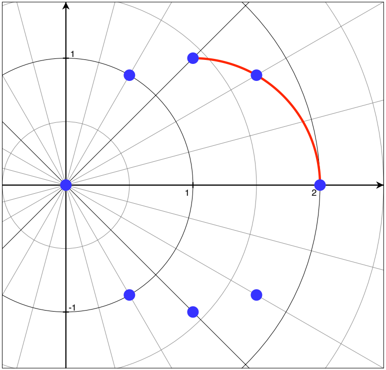

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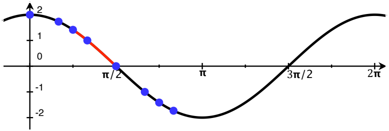

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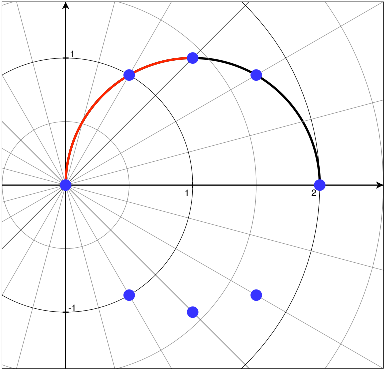

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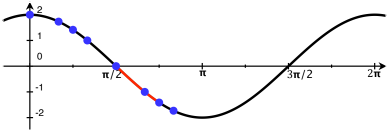

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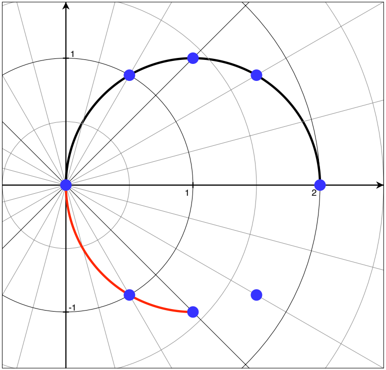

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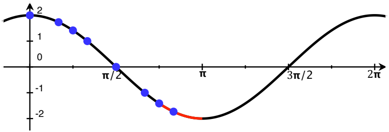

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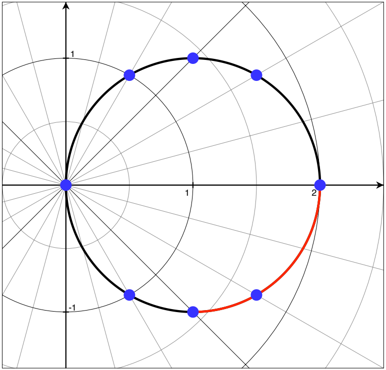

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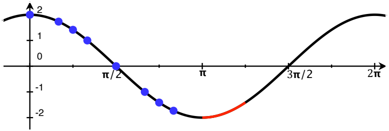

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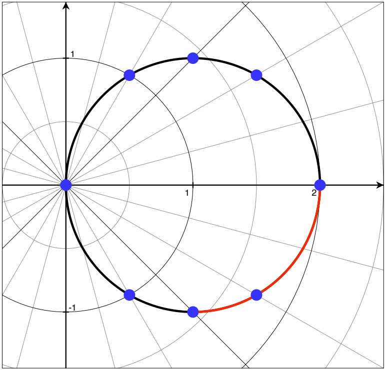

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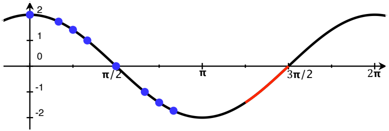

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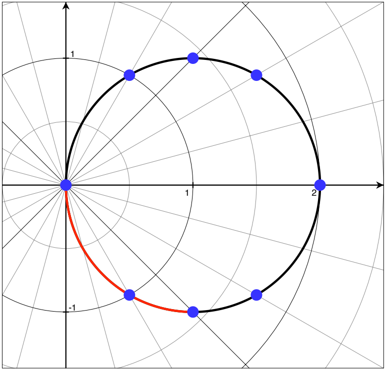
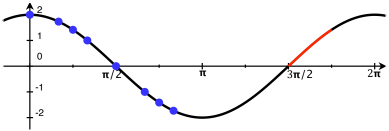


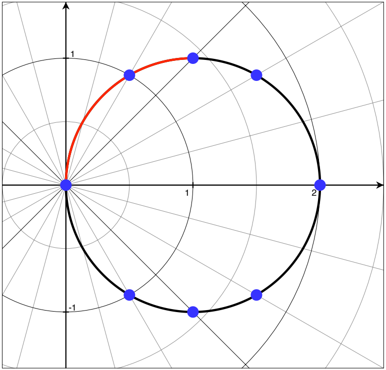
θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	2

Graph the function $r = 2\cos(\theta)$.

First, on a Cartesian (θ, r) plot, this function looks like



(1) Plot points, (2) piece together segments



θ	r
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$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
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π	2

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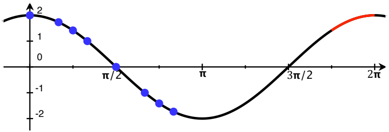
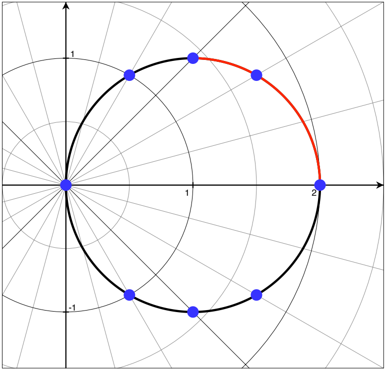


(1) Plot points, (2) piece together segments,
(3) stop at the end of one full period.



θ	r
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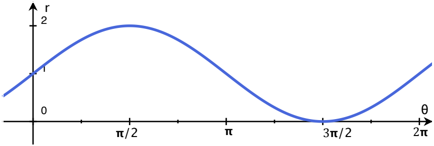
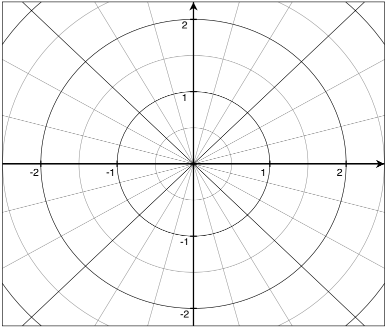
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	2

You try:

Note that on a θ-r axis, the curve $r = 1 + \sin(\theta)$ looks like



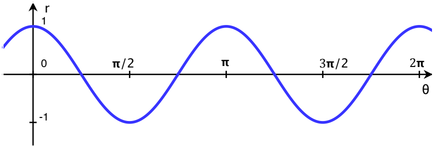
Sketch a graph of $r = 1 + \sin(\theta)$ on an x-y axis by plotting points, and piecing together segments as in the last example.



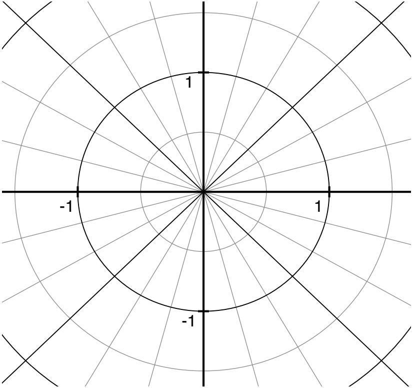
Note:
This graph is called the cardioid.

You try:

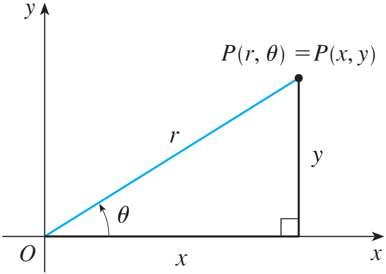
Note that on a θ-r axis, the curve $r = \cos(2\theta)$ looks like



Sketch a graph of $r = \cos(2\theta)$ on an x-y axis by plotting points, and piecing together segments as in the last example.



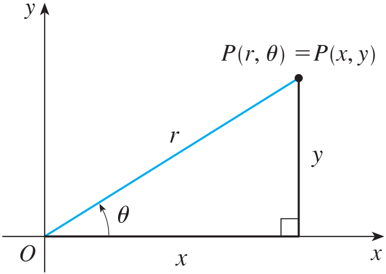
Converting between Cartesian and polar



The conversion from polar to cartesian comes the standard unit circle game, only now with radius r instead of 1:

$$\boxed{x = r\cos(\theta) \quad y = r\sin(\theta)}$$

Converting between Cartesian and polar



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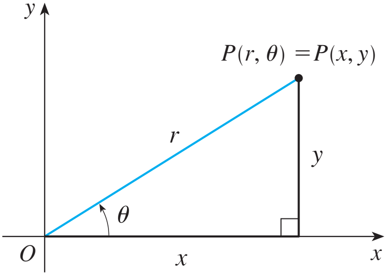
To get back, we must solve for r and θ.

Converting between Cartesian and polar



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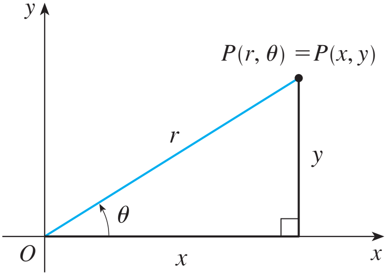
To get back, we must solve for r and θ. To solve for r, we have the Pythagorean identity:

$$x^2 + y^2 = (r\cos(\theta))^2 + (r\sin(\theta))$$

Converting between Cartesian and polar



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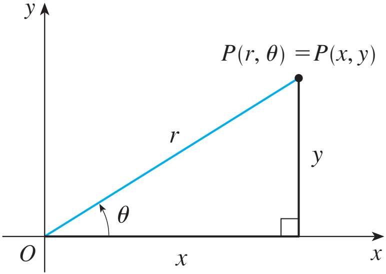

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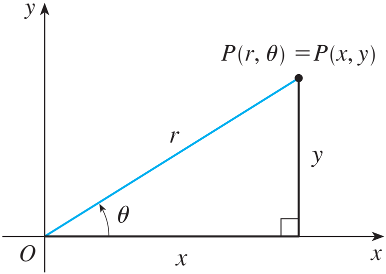

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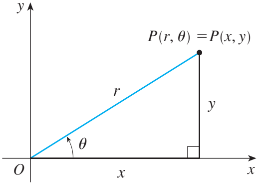
So

$$\boxed{r = \sqrt{x^2 + y^2}}.$$

Converting between Cartesian and polar



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To solve for θ, we eliminate r by dividing:

$$y/x = (r\sin(\theta))/(r\cos(\theta))$$

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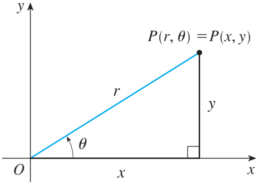

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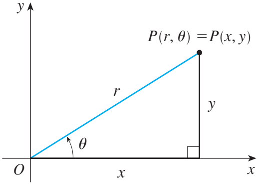
$$y/x = (r\sin(\theta))/(r\cos(\theta)) = \sin(\theta)/\cos(\theta)$$

Converting between Cartesian and polar



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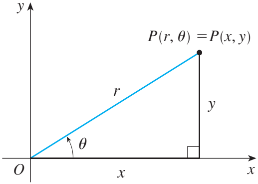
Converting between Cartesian and polar



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So

$$\boxed{\theta = \arctan(y/x)}.$$

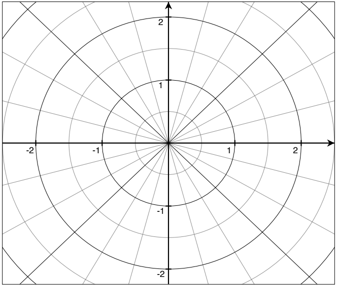
Converting between Cartesian and polar

$$\boxed{x = r\cos(\theta) \quad y = r\sin(\theta)}$$

$$\boxed{r = \sqrt{x^2 + y^2} \quad \theta = \arctan(y/x)}$$

You try: Convert the following points by filling out the rest of the table. Check by plotting.

(x, y)	(r, θ)
	$(2, \pi/3)$
	$(-1, \pi/4)$
$(1, 0)$	
$(1, -1)$	



Converting between Cartesian and polar

$$\boxed{x = r\cos(\theta) \quad y = r\sin(\theta)}$$

$$\boxed{r = \sqrt{x^2 + y^2} \quad \theta = \arctan(y/x)}$$

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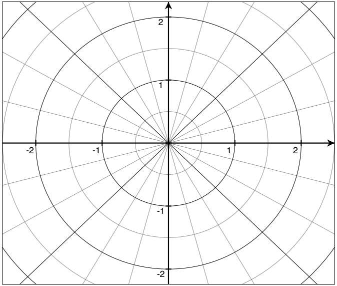
(x, y)	(r, θ)
$(1, \sqrt{3})$	$(2, \pi/3)$
$(-\sqrt{2}/2, -\sqrt{2}/2)$	$(-1, \pi/4)$
$(1, 0)$	$(1, 0)$
$(1, -1)$	$(\sqrt{2}, -\pi/4)$



Writing polar functions in terms of x and y

$$\boxed{x = r\cos(\theta) \quad y = r\sin(\theta)}$$

$$\boxed{r = \sqrt{x^2 + y^2} \quad \theta = \arctan(y/x)}$$

Example: $r = 2\cos(\theta)$.

Writing polar functions in terms of x and y

$$\boxed{x = r\cos(\theta) \quad y = r\sin(\theta)}$$

$$\boxed{r = \sqrt{x^2 + y^2} \quad \theta = \arctan(y/x)}$$

Example: $r = 2\cos(\theta)$. From $x = r\cos(\theta)$, we get $\cos(\theta) = x/r$.

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Example: $r = 2\cos(\theta)$. From $x = r\cos(\theta)$, we get $\cos(\theta) = x/r$. So

$$r = 2\cos(\theta) = 2x/r.$$

Writing polar functions in terms of x and y

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$$2x = (2x/r)r$$

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So

$2x = (2x/r)r = (r)r$

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So

$$r = 2\cos(\theta) = 2x/r.$$

So

$$2x = (2x/r)r = (r)r = r^2 = x^2 + y^2. \text{ Thus } 0 = x^2 - x + y^2.$$

Writing polar functions in terms of x and y

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$$r = 2\cos(\theta) = 2x/r.$$

So

$2x = (2x/r)r = (r)r = r^2 = x^2 + y^2$. Thus $0 = x^2 - x + y^2$.

Completing the square gives

$$(x-1)^2 + y^2 = 1,$$

which is a unit circle shifted right by 1, as we saw.

Writing polar functions in terms of x and y

$$\boxed{x = r\cos(\theta) \quad y = r\sin(\theta)}$$
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Completing the square gives

$$(x-1)^2 + y^2 = 1,$$

which is a unit circle shifted right by 1, as we saw.

You try: Write the following polar functions in terms of x and y.

(1) $r = 3$, (2) $\theta = \pi/3$, (3) $r = \sin(\theta)$.

Write the following Cartesian functions in terms of r and θ.

(1) $x^2 + y^2 = 4$, (2) $(x/3)^2 + y^2 = 1$, (3) $x = 2$.

Calculus with polar curves

Recall from last time, if I have a parametric curve $x = x(t)$, $y = y(t)$, then the slope of the line tangent to the curve plotted on an x-y axis is

$$\frac{dy}{dx} = \frac{dy}{dt} \bigg/ \frac{dx}{dt}.$$

Calculus with polar curves

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$$\frac{dy}{dx} = \frac{dy}{dt} \bigg/ \frac{dx}{dt}.$$

We can use this result if we think about a polar curve as a parametric curve in parameter θ:

$$r = r(\theta) \quad \longleftrightarrow \quad \begin{array}{c} x = r(\theta)\cos(\theta) \\ y = r(\theta)\sin(\theta) \end{array}$$

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Example: $r = e^{\theta}$ is the same as the parametric curve

$$x = e^{\theta}\cos(\theta), \quad y = e^{\theta}\sin(\theta).$$

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$$x = e^\theta \cos(\theta), \quad y = e^\theta \sin(\theta).$$

So now

$$\frac{dy}{dx} = \frac{dy}{d\theta} \bigg/ \frac{dx}{d\theta}$$

Calculus with polar curves

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We can use this result if we think about a polar curve as a parametric curve in parameter θ:

$$r = r(\theta) \quad \longleftrightarrow \quad \begin{aligned} x &= r(\theta)\cos(\theta) \\ y &= r(\theta)\sin(\theta) \end{aligned}$$

Example: $r = e^\theta$ is the same as the parametric curve

$$x = e^\theta \cos(\theta), \quad y = e^\theta \sin(\theta).$$

So now

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Bigg/ \frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))}$$

Calculus with polar curves

Recall from last time, if I have a parametric curve $x = x(t)$, $y = y(t)$, then the slope of the line tangent to the curve plotted on an x-y axis is

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$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))} = \frac{r'(\theta)\sin(\theta) + r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta) - r(\theta)\sin(\theta)}.$$

Calculus with polar curves

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Bigg/ \frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))} = \frac{r'(\theta)\sin(\theta) + r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta) - r(\theta)\sin(\theta)}.$$

Example: Let $r = 1 + \sin(\theta)$, the cardioid.

Calculus with polar curves

$$\boxed{\frac{dy}{dx} = \frac{dy}{d\theta}\bigg/\frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))} = \frac{r'(\theta)\sin(\theta) + r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta) - r(\theta)\sin(\theta)}.}$$

Example: Let $r = 1 + \sin(\theta)$, the cardioid. We have $\frac{dr}{d\theta} = \cos(\theta)$

Calculus with polar curves

$$\frac{dy}{dx} = \frac{dy}{d\theta} \bigg/ \frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))} = \frac{r'(\theta)\sin(\theta) + r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta) - r(\theta)\sin(\theta)}.$$

Example: Let $r = 1 + \sin(\theta)$, the cardioid. We have $\frac{dr}{d\theta} = \cos(\theta)$, so that

$$\frac{dy}{d\theta} = \cos(\theta)\sin(\theta) + (1 + \sin(\theta))\cos(\theta)$$

Calculus with polar curves

$$\boxed{\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))} = \frac{r'(\theta)\sin(\theta) + r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta) - r(\theta)\sin(\theta)}.}$$

Example: Let $r = 1 + \sin(\theta)$, the cardioid. We have $\frac{dr}{d\theta} = \cos(\theta)$, so that

$$\frac{dy}{d\theta} = \cos(\theta)\sin(\theta) + (1 + \sin(\theta))\cos(\theta) = \boxed{\cos(\theta)(1 + 2\sin(\theta))}$$

Calculus with polar curves

$$\boxed{\frac{dy}{dx} = \frac{dy}{d\theta}\Bigg/\frac{dx}{d\theta} = \frac{\frac{d}{d\theta}(r(\theta)\sin(\theta))}{\frac{d}{d\theta}(r(\theta)\cos(\theta))} = \frac{r'(\theta)\sin(\theta) + r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta) - r(\theta)\sin(\theta)}.}$$

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So $\boxed{\frac{dy}{dx} = \Big(\cos(\theta)(1 + 2\sin(\theta))\Big)/\Big(1 + \sin(\theta))(1 - 2\sin(\theta)\Big)}$

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So $\boxed{\frac{dy}{dx} = \Big(\cos(\theta)(1+2\sin(\theta))\Big)\Big/\Big(1+\sin(\theta))(1-2\sin(\theta)\Big)}$

You try: For what θ are the tangent lines to this cardioid horizontal? vertical?

$r = 1 + \sin(\theta)$