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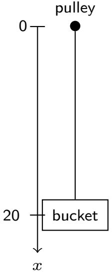

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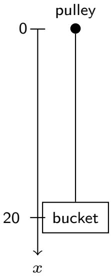

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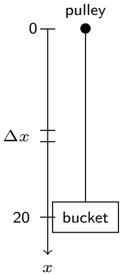

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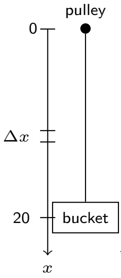

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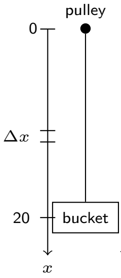

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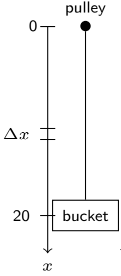

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$$W_{\text{water}} = \int_0^{20} 9 + \tfrac{1}{20}x \, dx = (9x + \tfrac{1}{40}x^2)\Big|_0^{20} = \boxed{9(20) + \tfrac{1}{40}(20)^2}.$$

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So in total,

$$W = W_{\text{bucket}} + W_{\text{rope}} + W_{\text{water}} = \boxed{2(20) + \tfrac{1}{4}(20)^2 + 9(20) + \tfrac{1}{40}(20)^2}.$$

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In the water portion of the previous problem, position and weight started out as functions of time:

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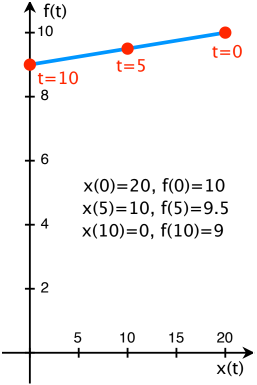

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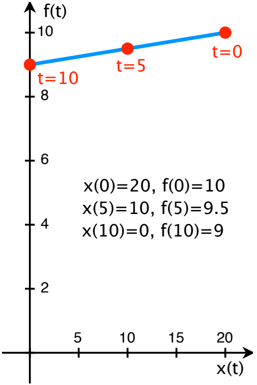

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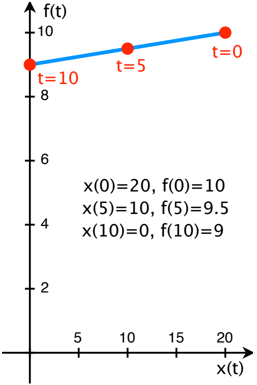

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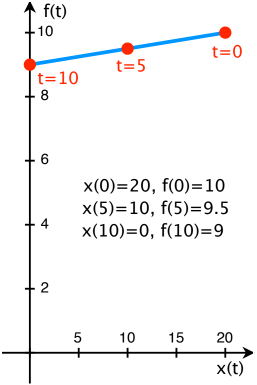

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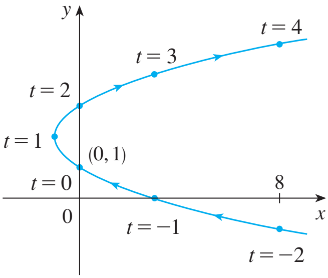

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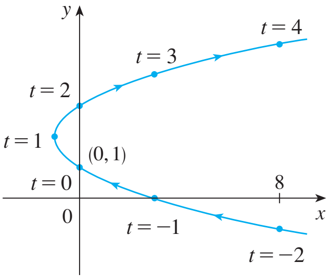

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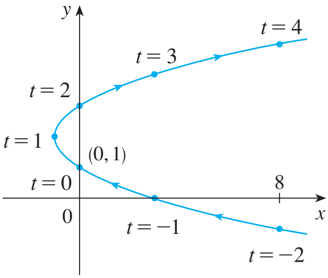

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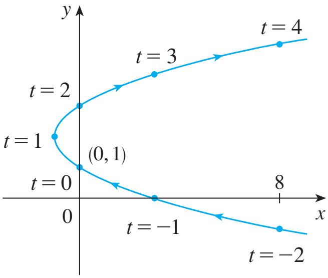

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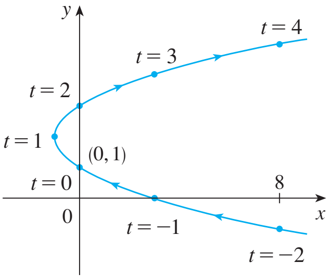

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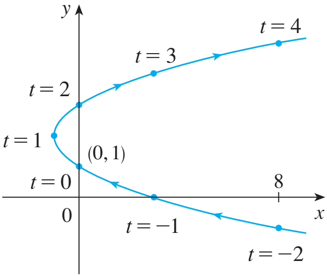

Writing the function just in terms of x and y loses some information.

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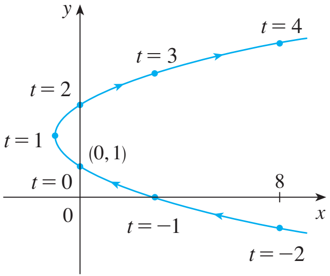

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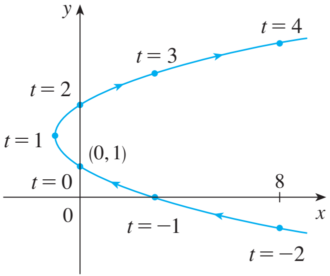

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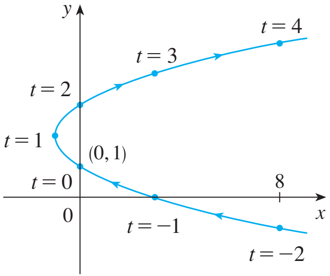

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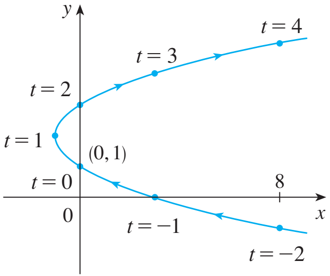

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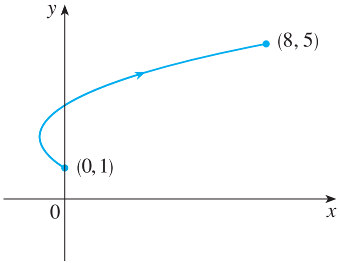

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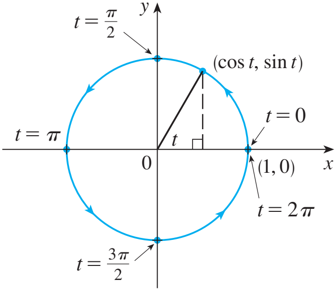

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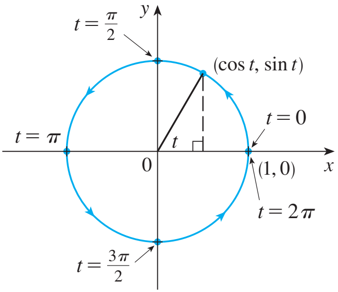

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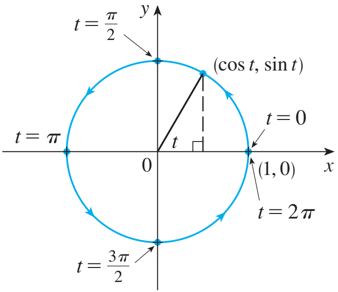

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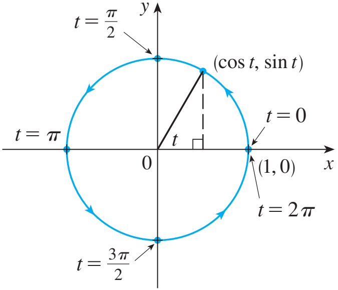

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Converting to a function of just x and y:

$$x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1.$$

You try: Graph and compare the following parametric curves to each other and the example above.

(1) $x(t) = \cos(2t), y(t) = \sin(2t), \quad 0 \leq t \leq 2\pi;$

(2) $x(t) = \cos(t/3), y(t) = \sin(t/3), \quad 0 \leq t \leq 2\pi.$

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Check:

$$(x - a)^2 + (y - b)^2 = (r\cos(t))^2 + (r\sin(t))^2 = r^2(\cos^2(t) + \sin^2(t)) = r^2. \checkmark$$

You try:

Sketch the following curves and give a formula for their shape just in terms of x and y.
(Hint: Think about graph transformations, and scaling or shifting either or both coordinates to get them to fit the pythagorean identity.) Give an example of a domain for t that would trace the curve exactly once.

1. $x(t) = 2\cos(t)$, $y(t) = \sin(t)$.
2. $x(t) = \cos(t)$, $y(t) = 3\sin(t)$.
3. $x(t) = \cos(t)$, $y(t) = \sin(-t)$.
4. $x(t) = 5\cos(2t)$, $y(t) = 3\sin(2t)$.
5. $x(t) = \cos(t)$, $y(t) = \sin(-t)$.
6. $x(t) = \sin(t)$, $y(t) = \cos(t)$.
7. $x(t) = 2\cos(t) + 1$, $y(t) = 3(\sin(t) - 4)$.
8. $x(t) = 5\cos(-t) + 1$, $y(t) = 2\sin(t) + 5$.

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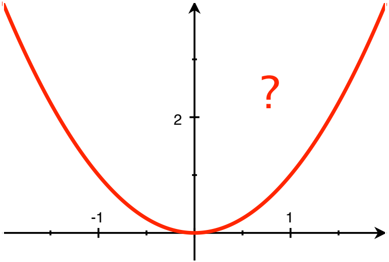

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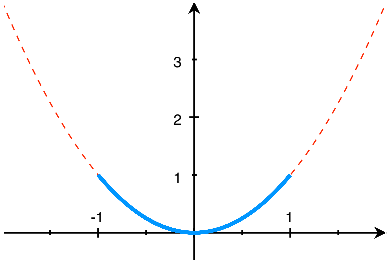

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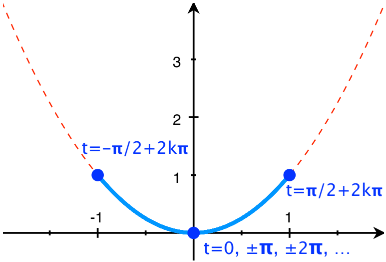
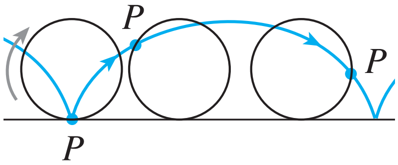


But $-1 \leq \sin(x) \leq 1$, so the x values can't go outside these bounds. This is actually a curve traced out by a particle bouncing back and forth between $(-1, 1)$ and $(1, 1)$ along the curve $y = x^2$.

Take a wheel or radius r, and mark one point on its boundary. Now roll that wheel, and trace the path that the marked point takes:



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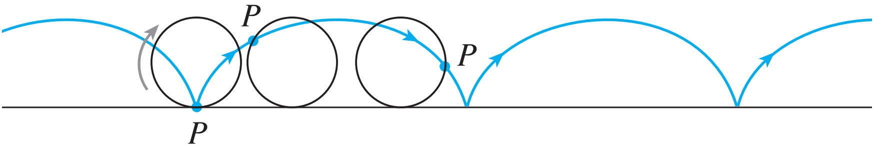

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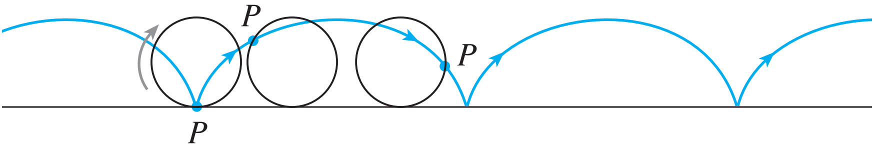

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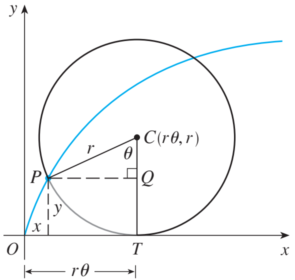

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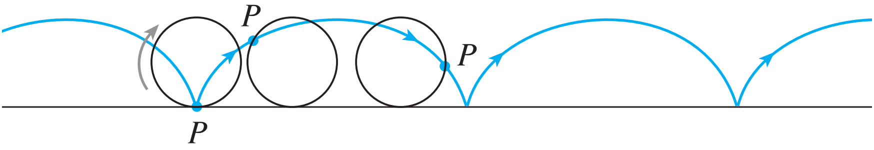

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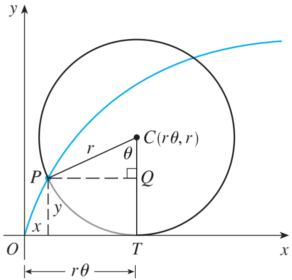

Parameter: θ, the rot'l angle of circle.

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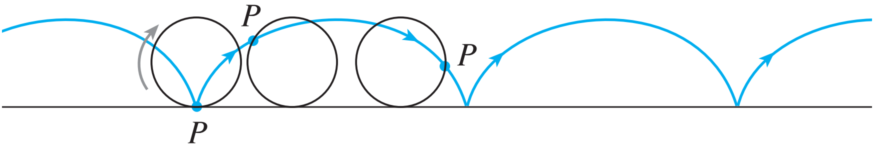

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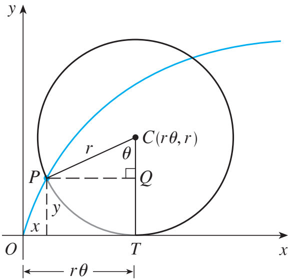

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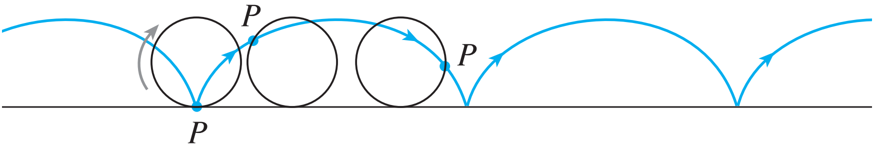

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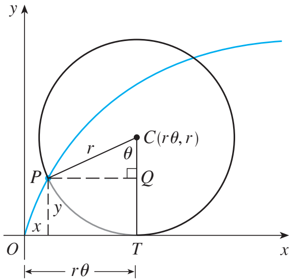

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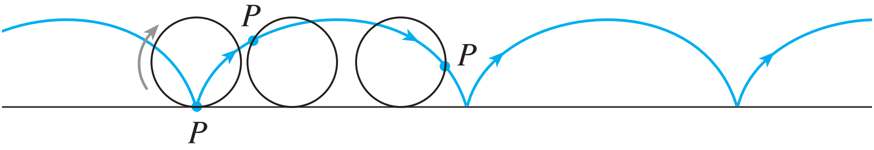

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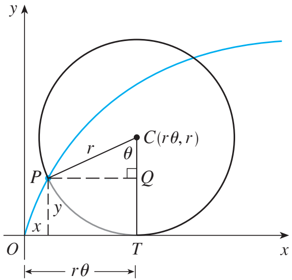

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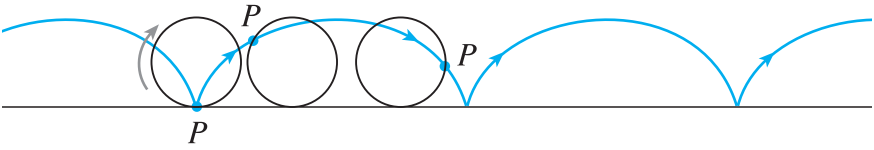

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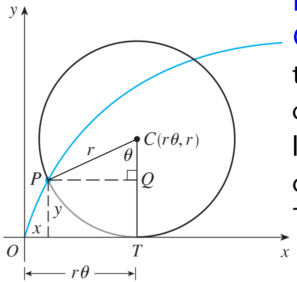

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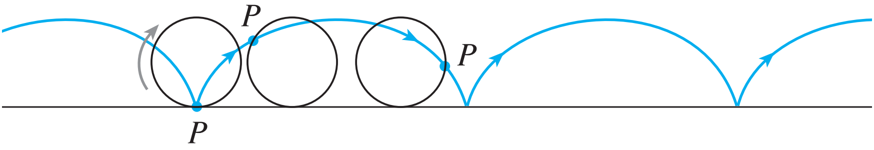

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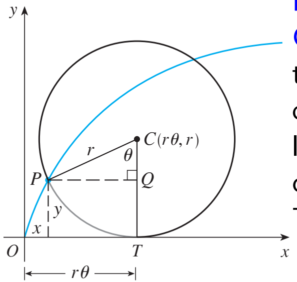

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