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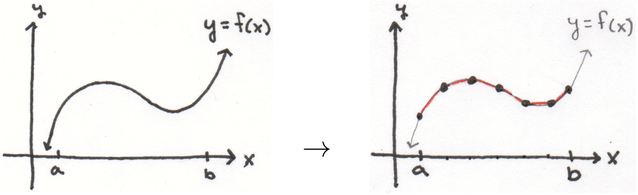
Today: instead of rotating filled in regions around an axis and calculating the volume of the shape, we will rotate curves around an axis and calculate the surface area of the shape.

The arc length approximation was a lot like when we approximated area with trapezoids, where each piece is a line with some slope, instead of constant lines



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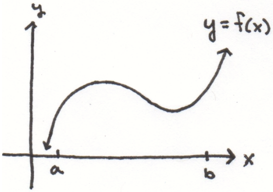

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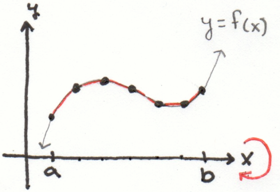

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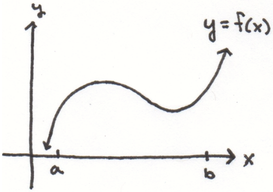

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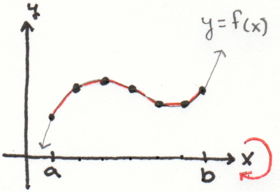

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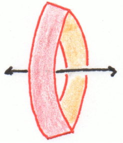

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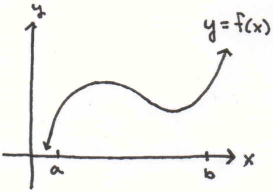

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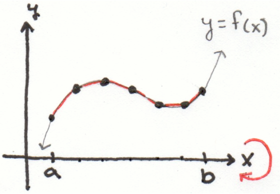

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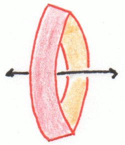
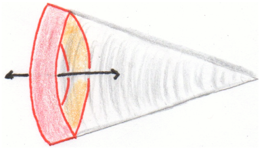
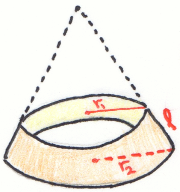


, which is part of the cone



.

So we need the surface area of a cross-section of a cone,



which depends on the lateral length ℓ of the slice, and the average of the radius r_1 of the small circle and the radius r_2 of the big circle.

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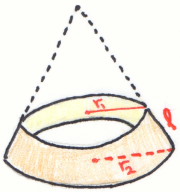

which depends on the lateral length ℓ of the slice, and the average of the radius r_1 of the small circle and the radius r_2 of the big circle. This surface area is given by (see section 7.5)

$$A = 2\pi \left(\frac{1}{2}(r_1 + r_2)\right) \ell$$

(i.e. the area of the circular cylinder whose height is the the lateral length ℓ and whose radius is the average of the two extreme radii).

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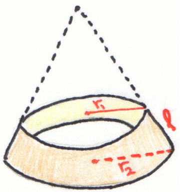

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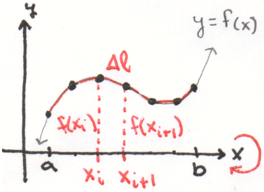

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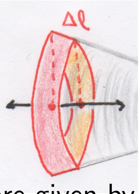

the lateral length is $\Delta\ell$ and the two radii are given by the height of the function at x_i and x_{i+1}.

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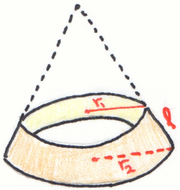

$$A = 2\pi\left(\tfrac{1}{2}(r_1 + r_2)\right)\ell$$
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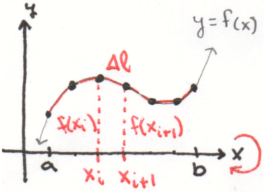

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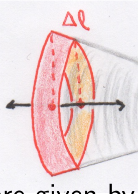

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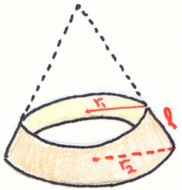

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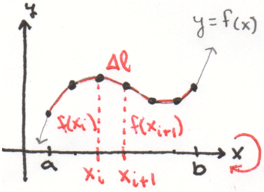

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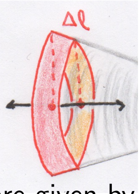

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$\Delta\ell \to d\ell = \sqrt{1 + (\frac{dy}{dx})^2}\, dx$ and $\frac{1}{2}(f(x_i) + f(x_{i+1})) \to f(x)$.

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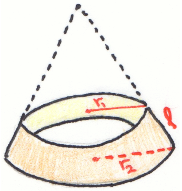

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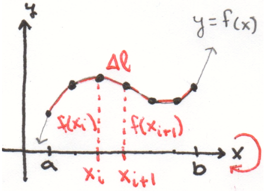

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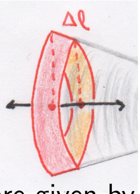

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Example

$$\boxed{dA = 2\pi f(x)\sqrt{1+(f'(x))^2}\ dx}$$

Example

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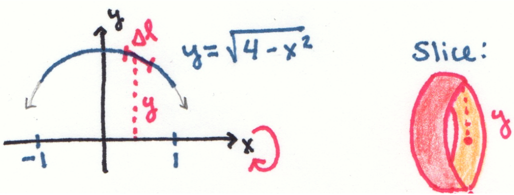
Calculate the surface area of shape generated by rotating the curve $y = \sqrt{4-x^2}$, $-1 \le x \le 1$, around the x-axis:



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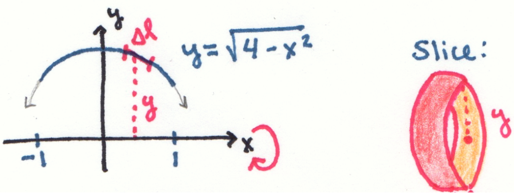

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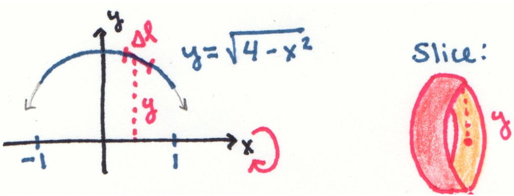

Here, $f(x) = \sqrt{4 - x^2}$, so that $f(x) = \frac{-2x}{2\sqrt{4-x^2}}$

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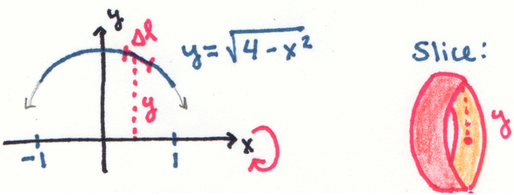

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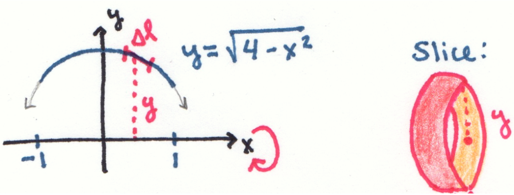

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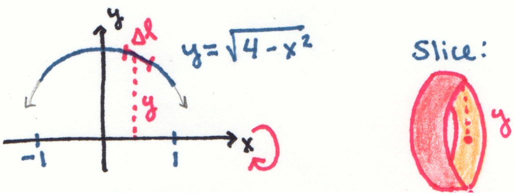

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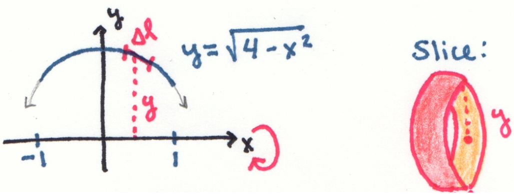

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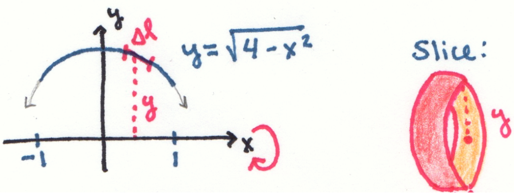

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So

$$A = \int_{x=-1}^{1} dA$$

Example

$$\boxed{dA = 2\pi f(x)\sqrt{1+(f'(x))^2}\ dx}$$

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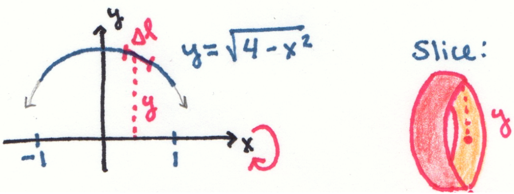

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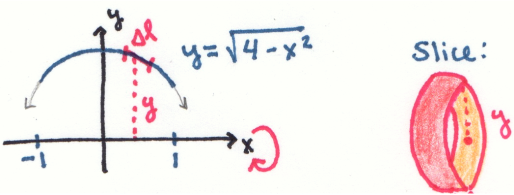

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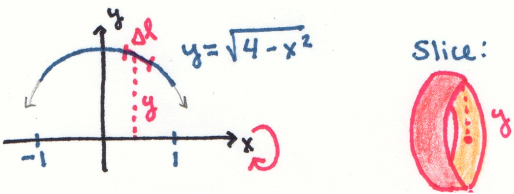

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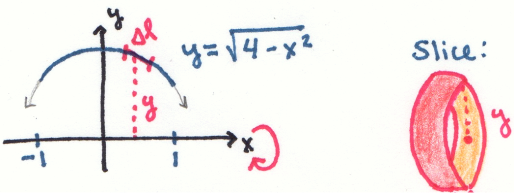

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You try:

Set up the following problems (simplify until you have something you can integrate, but don't finish the integration).

Calculate the surface area of shape resulting from...

1. revolving $y = \sqrt{1 + e^x}$ for $0 \le x \le 1$ around the x-axis;
2. revolving $y = 1 + 2x^2$ for $1 \le x \le 2$ around the x-axis;
3. revolving $y = \sin(x)$ for $0 \le x \le \pi/4$ around the x-axis.

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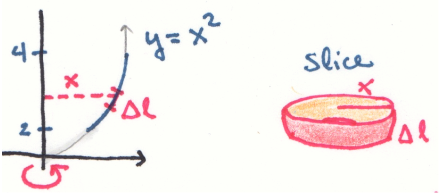

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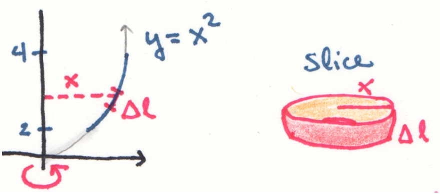

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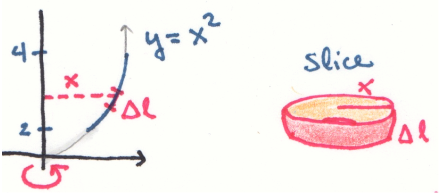

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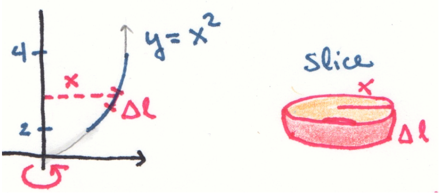

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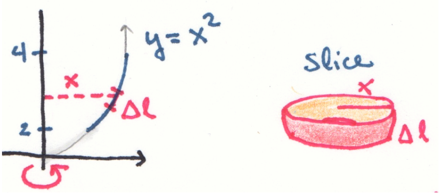

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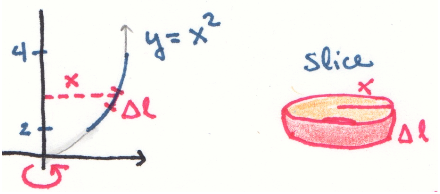

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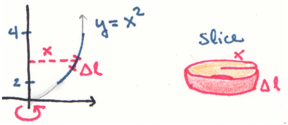

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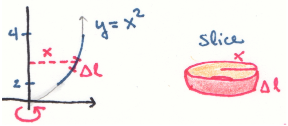

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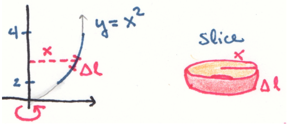

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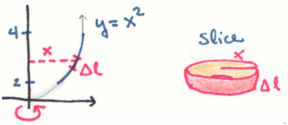

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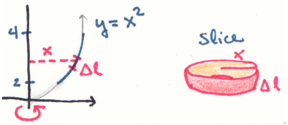

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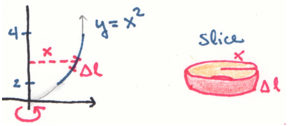

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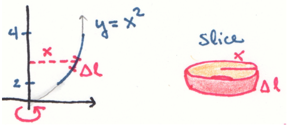

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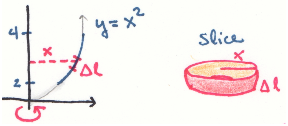

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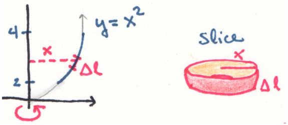

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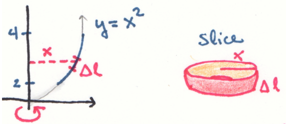

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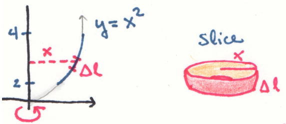

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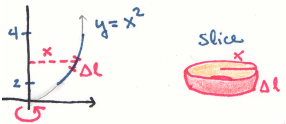

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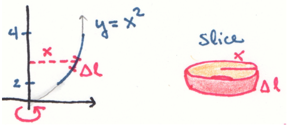

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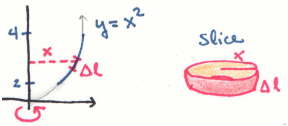

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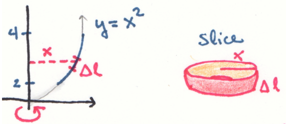

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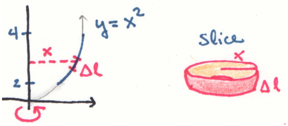

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You try:

Consider the shape generated by revolving

$$y = \frac{1}{2}\sqrt{x-1} \quad 3 \leq x \leq 9$$

around the x-axis. Compute (if possible) the resulting surface area in two ways, using x as the variable, and then using y as the variable. Then do the same thing for rotating the same curve around the y-axis.

Suggestion for studying:

Make an outline of the measurements we've made, and the formulas for the slices.

- Area (flat)
- Volume
- Length
- Area (curved)

Categorize the cases (axes of revolution, etc) and how those cases change the formulas for the slices.

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Example: Lifting a 1.2 kg book takes a force of mg, where $g = 9.8$ m/s^2 is the acceleration of gravity. So lifting the book 0.7 m off the ground takes an effort of

$$W = (1.2 \text{ kg})(9.8 \text{ m/s}^2)(0.7 \text{ m}) = (1.2)(9.8)(0.1) \text{ J},$$

where J is joules (kg $*$ m^2 / s^2).

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7.6 "Work" from physics

In physics, work W means the the effort it takes to move an object. This is the amount of force exerted, times the distance d moved. Newton's second law of motion describes force F as mass m times acceleration a: $F = ma$. But acceleration is the second derivative of position $s(t)$ versus time t. So

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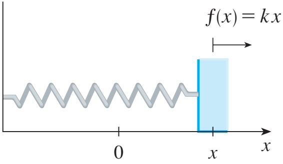
$$F = kx, \quad \text{where } k \text{ is constant, depending on the spring.}$$



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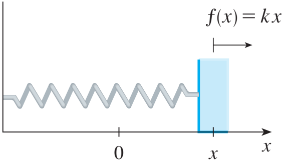

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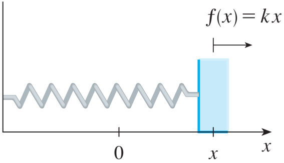

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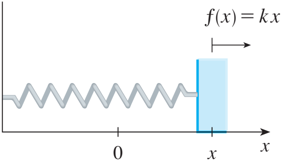

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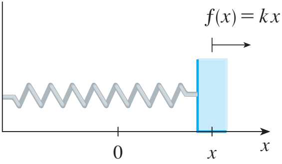

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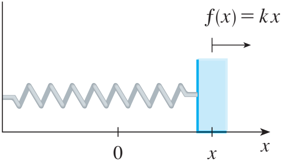

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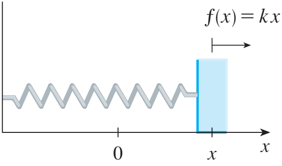

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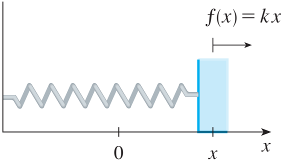

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