Today: 6.1 Integration by parts

Warm up: Differentiate the following functions

(1) $x \ln(x)$, (2) $\cosh(x) \sinh(x)$, (3) $\cos(x) \sin(x)$,

(4) $\sqrt{x} \sinh^{-1}(x)$, (5) $3^x x^3$, (6) $(1 + x^2) \tan^{-1}(x)$,

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Answers:

(1) $\ln(x)+1$, (2) $\sinh^2(x)+\cosh^2(x)$, (3) $\cos^2(x)-\sin^2(x)$,

(4) $\sinh^{-1}(x)/2\sqrt{x}+\sqrt{x}/\sqrt{x^2+1}$

(5) $3^x x^2(\ln(3)x+3)$, (6) $2x\tan^{-1}(x)+1$

Integration by parts: undoing product rule

Recall, product rule:

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

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How this helps: Calculate

$$\int x \cos(x)\, dx.$$

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$$x\sin(x) = \int 1 \cdot \sin(x)\, dx + \int x\cos(x)\, dx$$

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$$x\sin(x) = \int 1 \cdot \sin(x) \, dx + \int x\cos(x) \, dx = -\cos(x) + \int x\cos(x) \, dx.$$

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$$x\sin(x) = \int 1 \cdot \sin(x)\, dx + \int x\cos(x)\, dx = -\cos(x) + \int x\cos(x)\, dx.$$

So $\int x\cos(x)\, dx = x\sin(x) + \cos(x)$.

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Integrating both sides of $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ gives

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Start by picking

$f(x)$ so that (usually) $f'(x)$ is simpler than $f(x)$,

and $g'(x)$ so that we can compute $\int g'(x)\ dx$.

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$f'(x)$:	$g'(x)$: $\cos(x)$

Then compute $f'(x)$ and $g(x)$.

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Then compute $f'(x)$ and $g(x)$.

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$f(x)$:	$g(x)$:
x	$\sin(x)$
$f'(x)$:	$g'(x)$:
1	$\cos(x)$

Then compute $f'(x)$ and $g(x)$.

Finally, plug in and hope we're left with something we can compute:

$$\int x\cos(x)\, dx = x \cdot \sin(x) - \int 1 \cdot \cos(x)\, dx \ \checkmark$$

You try

$$\boxed{\int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)\ dx.}$$

Compute the integral

$$\int xe^x\ dx$$

by letting

$$f(x) = x \quad \text{and} \quad g'(x) = e^x.$$

(Compute $f'(x)$ and $g(x) = \int g'(x)\ dx$, and then plug into the formula)

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Answer:

$$f'(x) = 1 \qquad \text{and} \qquad g(x) = \int e^x\ dx = e^x \text{ (leave off } +C\text{)}$$

So

$$\int xe^x\ dx = xe^x - \int 1 \cdot e^x\ dx = xe^x - e^x + C.$$

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Check: $\quad \dfrac{d}{dx}(xe^x + e^x) = e^x + xe^x - e^x = xe^x$ ✓

You try: Do it twice

$$\boxed{\int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)\ dx.}$$

Compute $\displaystyle\int x^2 \cos(x)\ dx.$

Start by letting

$$f(x) = x^2 \quad \text{and} \quad g'(x) = \cos(x).$$

When you get to computing $\int f'(x)g(x)\ dx$, you'll need to use integration by parts again.

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Answer: $(x^2-2)\sin(x) + 2x\cos(x) + C$

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Answer: $(x^2 - 2)\sin(x) + 2x\cos(x) + C$

A note on notation: In the book, they use u and dv in place of $f(x)$ and $g'(x)$. You can use either.

You try: Something out of nothing

$$\boxed{\int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)\ dx.}$$

Compute

$$\int \ln(x)\ dx$$

using integration by parts by letting

$$f(x) = \ln(x) \quad \text{and} \quad g'(x) = 1.$$

Note: This is a common technique for computing the integral of inverse functions, like $\sin^{-1}(x)$, $\tan^{-1}(x)$, $\sinh^{-1}(x)$, etc..

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Answer: $x(\ln(x) - 1) + C$

Note: This is a common technique for computing the integral of inverse functions, like $\sin^{-1}(x)$, $\tan^{-1}(x)$, $\sinh^{-1}(x)$, etc..

Something else out of nothing, u-sub edition

Compute $$\int e^{\sqrt{x}}\,dx.$$

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$$\text{Compute} \quad \int e^{\sqrt{x}}\, dx.$$

We could try letting $f(x) = e^{\sqrt{x}}$ and $g'(x) = 1$, like before.

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We could try letting $f(x) = e^{\sqrt{x}}$ and $g'(x) = 1$, like before.
So $f'(x) = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}$ and $g(x) = x$.

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We could try letting $f(x) = e^{\sqrt{x}}$ and $g'(x) = 1$, like before.
So $f'(x) = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}$ and $g(x) = x$. Thus

$$\int e^{\sqrt{x}}\, dx = xe^{\sqrt{x}} - \tfrac{1}{2} \underbrace{\int \sqrt{x} e^{\sqrt{x}}\, dx}_{\text{not a lot better...}} .$$

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Instead, we can start with a u-substitution:
Let $u = \sqrt{x}$.

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Instead, we can start with a u-substitution:
Let $u = \sqrt{x}$. So $du = \frac{1}{2\sqrt{x}} dx = \frac{1}{2u} dx$.

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So $f'(x) = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}$ and $g(x) = x$. Thus

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Instead, we can start with a u-substitution:
Let $u = \sqrt{x}$. So $du = \frac{1}{2\sqrt{x}} dx = \frac{1}{2u} dx$.
So $2u\, du = dx$, giving

$$\int e^{\sqrt{x}}\, dx = \int 2ue^{u} du.$$

Finish using integration by parts like before!

A cyclic example: Compute $\int e^x \cos(x)\ dx$.

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A cyclic example: Compute $\int e^x \cos(x)\ dx$.

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Thus

$$\int e^x \cos(x)\ dx = e^x \sin(x) - \int e^x \sin(x)\ dx.$$

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Now, to compute $\int e^x \sin(x)\ dx$:

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Now, to compute $\int e^x \sin(x)\ dx$:
Let $f(x) = e^x$ and $g'(x) = \sin(x)$.
So $f'(x) = e^x$ and $g(x) = -\cos(x)$.

A cyclic example: Compute $\int e^x \cos(x)\ dx$.

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Thus

$$\int e^x \sin(x)\ dx = -e^x \cos(x) - \int e^x(-\cos(x))\ dx$$

A cyclic example: Compute $\int e^x \cos(x)\, dx$.

Let $f(x) = e^x$ and $g'(x) = \cos(x)$.
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Thus

$$\int e^x \cos(x)\, dx = e^x \sin(x) - \int e^x \sin(x)\, dx.$$

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Thus

$$\int e^x \sin(x)\, dx = -e^x \cos(x) - \int e^x(-\cos(x))\, dx$$

Putting these together, we have

$$\int e^x \cos(x)\, dx = e^x \sin(x) - \left(-e^x \cos(x) + \int e^x \cos(x)\right).$$

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Thus

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Putting these together, we have

$$\int e^x \cos(x)\ dx = e^x \sin(x) - \left(-e^x \cos(x) + \int e^x \cos(x)\right).$$

So $$2\int e^x \cos(x)\ dx = e^x \sin(x) + e^x \cos(x) + C.$$

A cyclic example: Compute $\int e^x \cos(x)\ dx$.

Let $f(x) = e^x$ and $g'(x) = \cos(x)$. So $f'(x) = e^x$ and $g(x) = \sin(x)$.

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Putting these together, we have

$$\int e^x \cos(x)\ dx = e^x \sin(x) - \left(-e^x \cos(x) + \int e^x \cos(x)\right).$$

So $$2\int e^x \cos(x)\ dx = e^x \sin(x) + e^x \cos(x) + C.$$

And thus

$$\int e^x \cos(x)\ dx = \tfrac{1}{2} e^x(\sin(x) + \cos(x)) + D.$$

You try:

Compute the following antiderivatives using integration by parts.

1. $\int x \sec^2(x)\ dx$
2. $\int \sin^{-1}(x)\ dx$
3. $\int x \ln(x)\ dx$

   (Try first by letting $f(x) = x$, and try again letting $f(x) = \ln(x)$)
4. $\int \cos^2(x)\ dx$

   (Try letting $f(x) = \cos(x)$ and $g'(x) = \cos(x)$, and then using $\sin^2(x) = 1 - \cos^2(x)$. Then look back at the $e^x \cos(x)$ example.)

You try:

Compute the following antiderivatives using integration by parts.

1. $\int x \sec^2(x)\ dx$ Answer: $x\tan(x) + \ln(\cos(x)) + C$
2. $\int \sin^{-1}(x)\ dx$ Answer: $x\sin^{-1}(x) + \sqrt{1-x^2} + C$
3. $\int x\ln(x)\ dx$ Answer: $x^2(2\ln(x) - 1)/4 + C$

   (Try first by letting $f(x) = x$, and try again letting $f(x) = \ln(x)$)
4. $\int \cos^2(x)\ dx$ Answer: $\frac{1}{2}(x + \sin(x)\cos(x)) + C$

   (Try letting $f(x) = \cos(x)$ and $g'(x) = \cos(x)$, and then using $\sin^2(x) = 1 - \cos^2(x)$. Then look back at the $e^x\cos(x)$ example.)