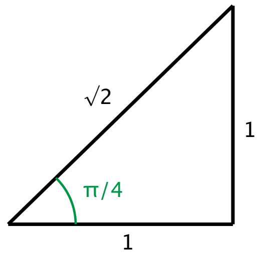
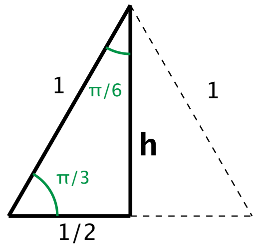
Warm up: Fill out the following tables.

isosceles right triangle:



equilateral triangle cut in half:

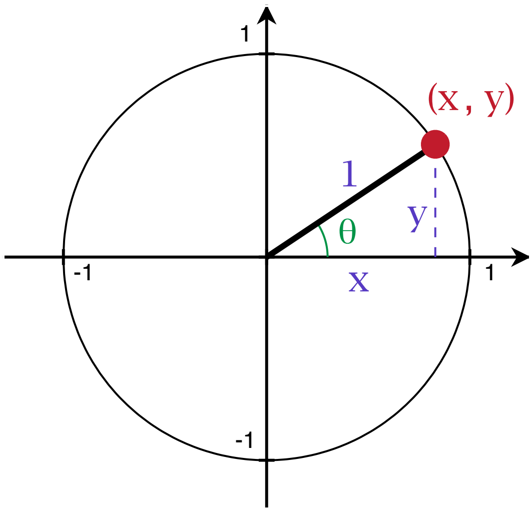


$$h = \sqrt{1 - (1/2)^2} = \sqrt{3}/2$$

	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\pi/4$						
$\pi/3$						
$\pi/6$						

f(x)	$\tan(x)$	$\sec(x)$	$\csc(x)$	$\cot(x)$
f'(x)				

Reviewing the unit circle



For $0 < \theta < \frac{\pi}{2}$...

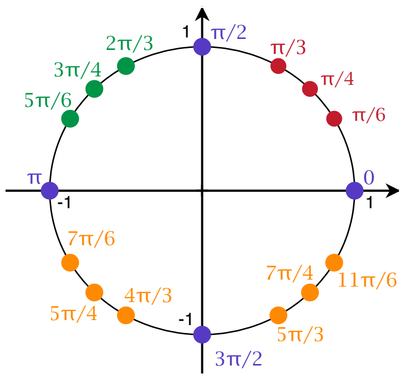
$$\cos(\theta) = \frac{x}{1} = x$$

$$\sin(\theta) = \frac{y}{1} = y$$

Use this idea to extend trig functions to any θ, defining

$$\cos(\theta) = x \qquad \sin(\theta) = y.$$

Review: what we can read off of the unit circle



$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta)$$

$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$

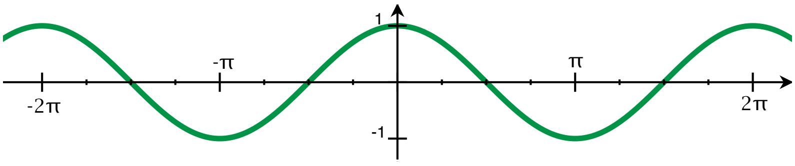
$$x^2 + y^2 = 1 \implies \cos^2(\theta) + \sin^2(\theta) = 1$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\sin(\theta)$	0	1	0	-1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$

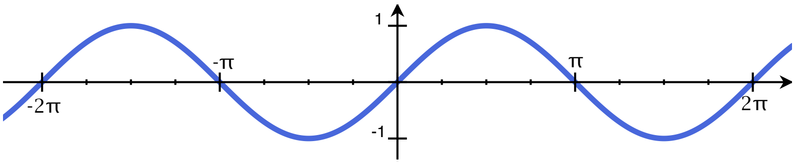
	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$

Plotting on the θ-y axis

Graph of $y = \cos(\theta)$:



Graph of $y = \sin(\theta)$:



Trig identities to know and love:

Even/odd:

$\cos(-\theta) = \cos(\theta)$ (even) $\quad \sin(-\theta) = -\sin(\theta)$ (odd)

Pythagorean identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

Angle addition:

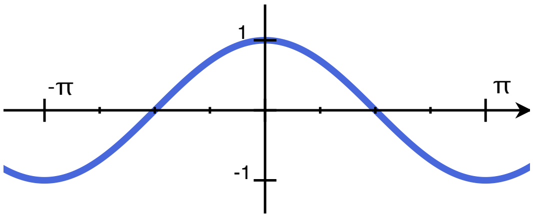
$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

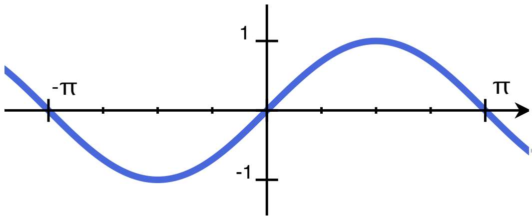
(in particular $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ *and* $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$*)*

Other trig functions

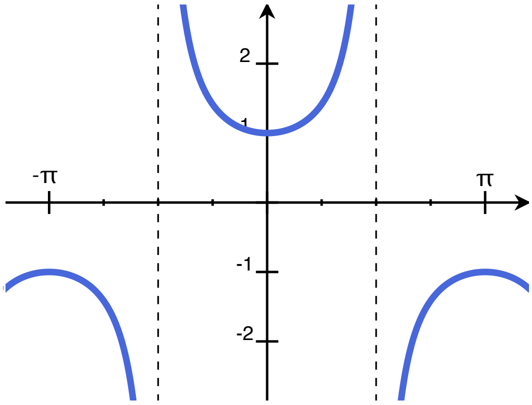
$y = \cos(\theta)$



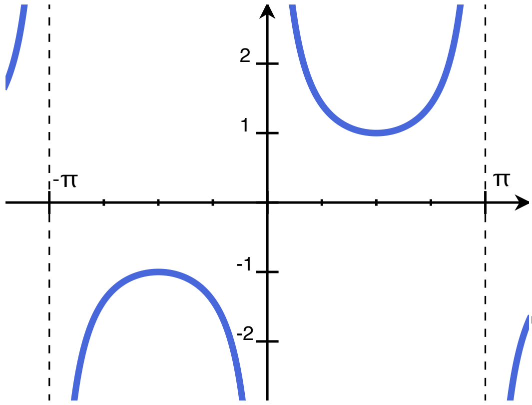
$y = \sin(\theta)$



$\sec(\theta) = 1/\cos(\theta)$

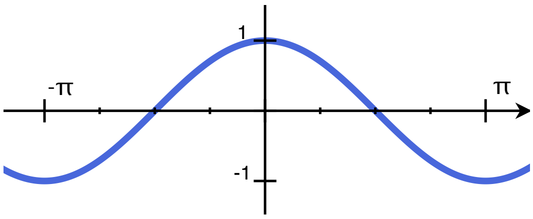


$\csc(\theta) = 1/\sin(\theta)$

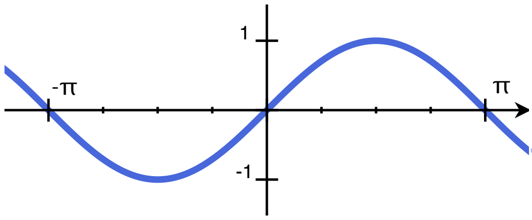


Other trig functions

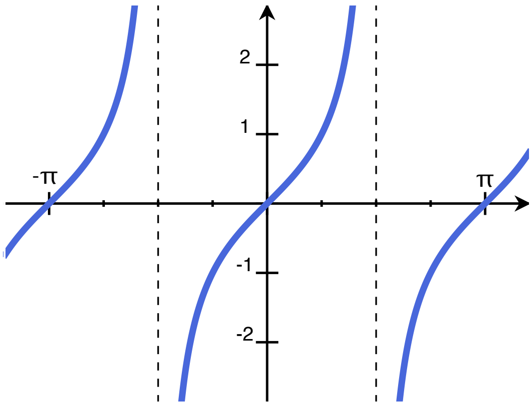
$y = \cos(\theta)$



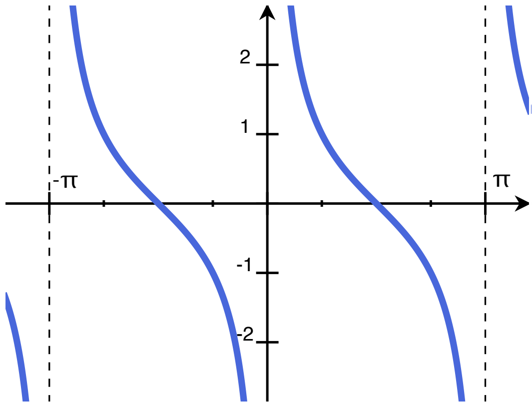
$y = \sin(\theta)$



$\tan(\theta) = \sin(\theta)/\cos(\theta)$



$\cot(\theta) = \cos(\theta)/\sin(\theta)$



Inverse trig functions

Two notations:

$f(x)$	$f^{-1}(x)$
$\sin(x)$	$\sin^{-1}(x) = \arcsin(x)$
$\cos(x)$	$\cos^{-1}(x) = \arccos(x)$
$\tan(x)$	$\tan^{-1}(x) = \arctan(x)$
$\sec(x)$	$\sec^{-1}(x) = \text{arcsec}(x)$
$\csc(x)$	$\csc^{-1}(x) = \text{arccsc}(x)$
$\cot(x)$	$\cot^{-1}(x) = \text{arccot}(x)$

There are lots of points we know on these functions...

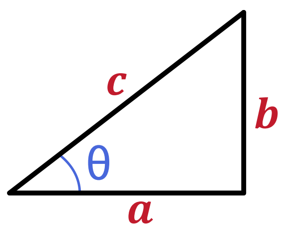
Examples:

1. Since $\sin(\pi/2) = 1$, we have $\arcsin(1) = \pi/2$
2. Since $\cos(\pi/2) = 0$, we have $\arccos(0) = \pi/2$

Etc...

In general:

arc___(-) takes in a ratio and spits out an angle:



$$\cos(\theta) = a/c \quad \text{so} \quad \arccos(a/c) = \theta$$

$$\sin(\theta) = b/c \quad \text{so} \quad \arcsin(b/c) = \theta$$

$$\tan(\theta) = b/a \quad \text{so} \quad \arctan(b/a) = \theta$$

Domain problems:

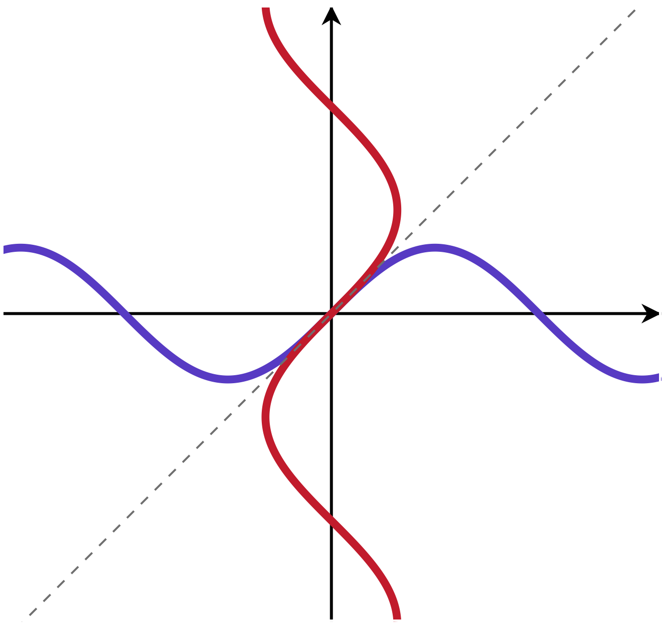
$$\sin(0) = 0, \qquad \sin(\pi) = 0, \qquad \sin(2\pi) = 0, \qquad \sin(3\pi) = 0, \ldots$$

So which is the right answer to $\arcsin(0)$, really?

Domain/range

$y = \sin(x)$

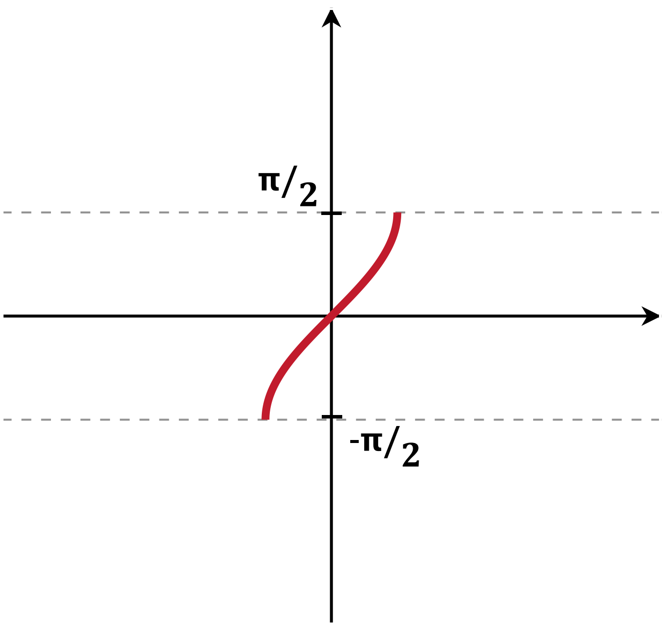
$y = \arcsin(x)$



Domain: $-1 \leq x \leq 1$

Domain/range

$y = \arcsin(x)$

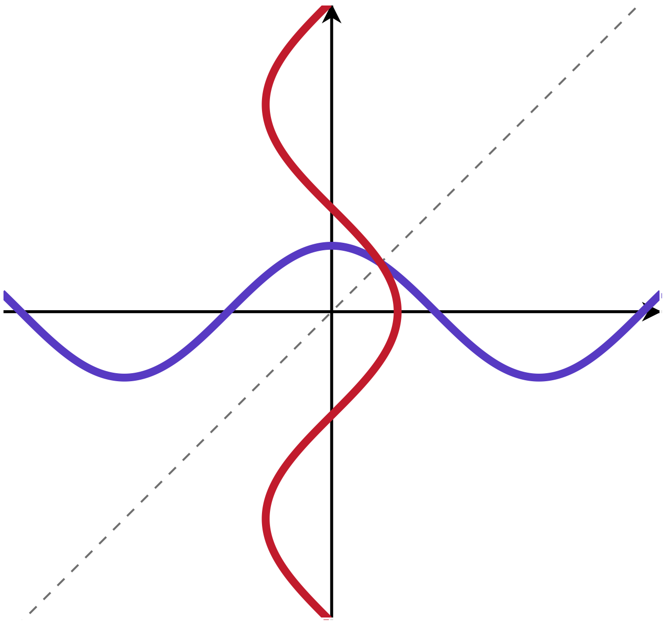


Domain: $-1 \leq x \leq 1$ Range: $-\pi/2 \leq y \leq \pi/2$

Domain/range

$y = \cos(x)$

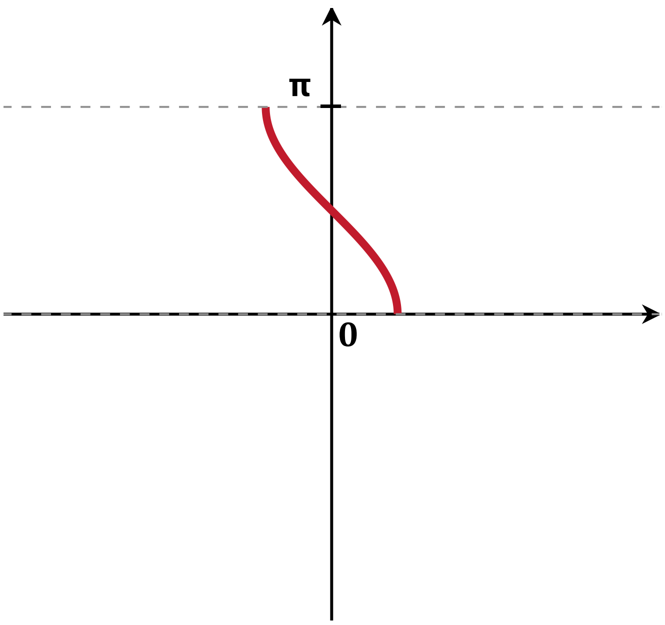
$y = \arccos(x)$



Domain: $-1 \leq x \leq 1$

Domain/range

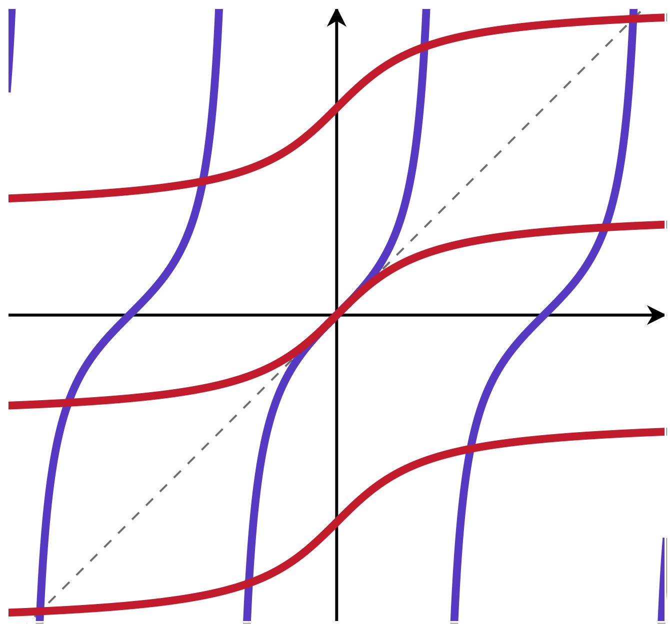
$y = \arccos(x)$



Domain: $-1 \leq x \leq 1$ Range: $0 \leq y \leq \pi$

Domain/range

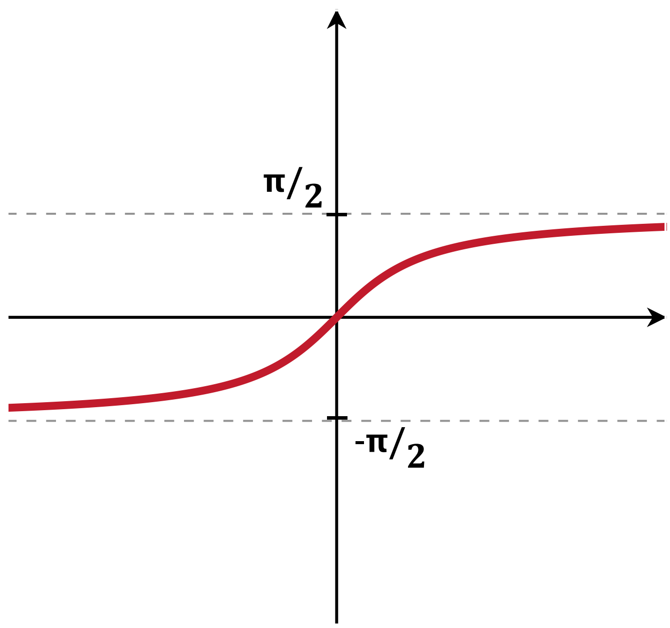
$y = \tan(x)$
$y = \arctan(x)$



Domain: $-\infty \leq x \leq \infty$

Domain/range

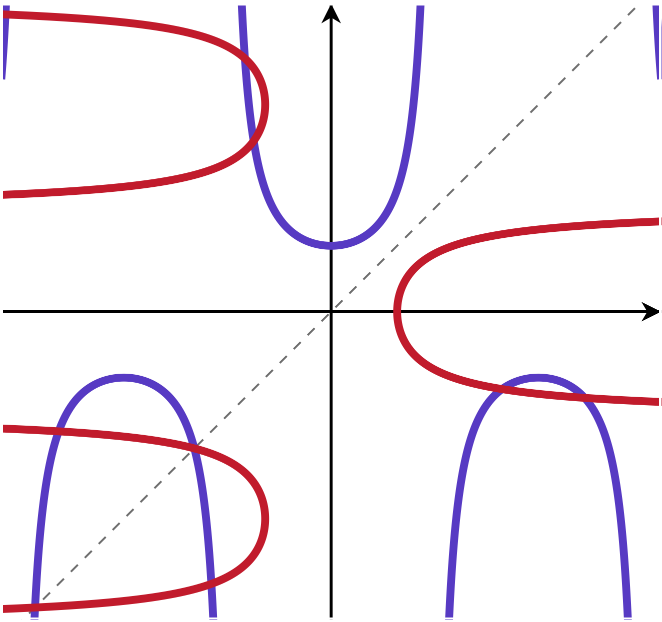
$y = \arctan(x)$



Domain: $-\infty \leq x \leq \infty$ Range: $-\pi/2 < y < \pi/2$

Domain/range

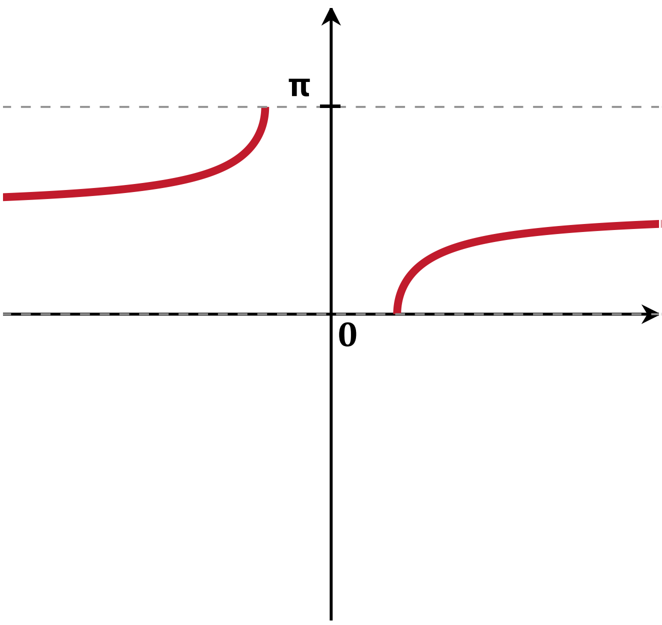
$y = \sec(x)$
$y = \text{arcsec}(x)$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

$y = \text{arcsec}(x)$

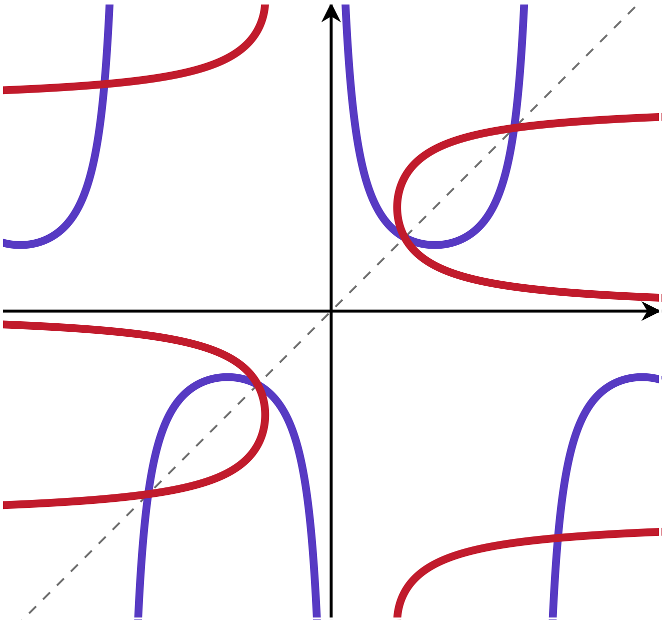


Domain: $x \leq -1$ and $1 \leq x$ Range: $0 \leq y \leq \pi$

Domain/range

$y = \csc(x)$

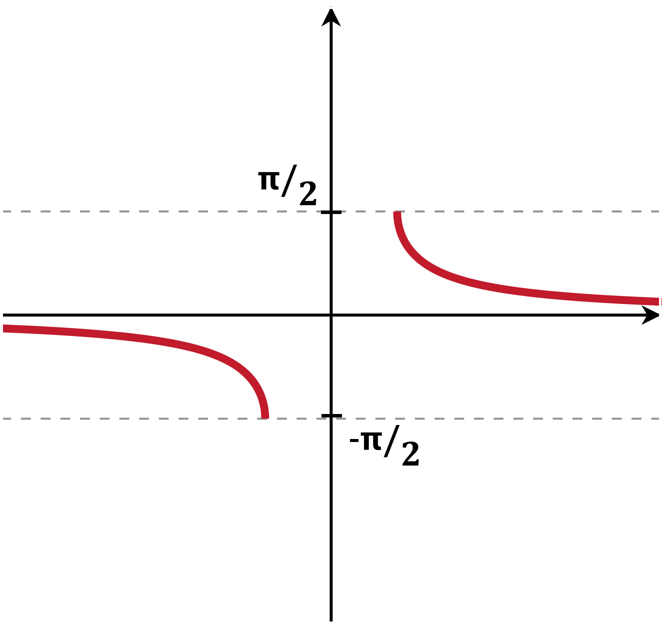
$y = \text{arccsc}(x)$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

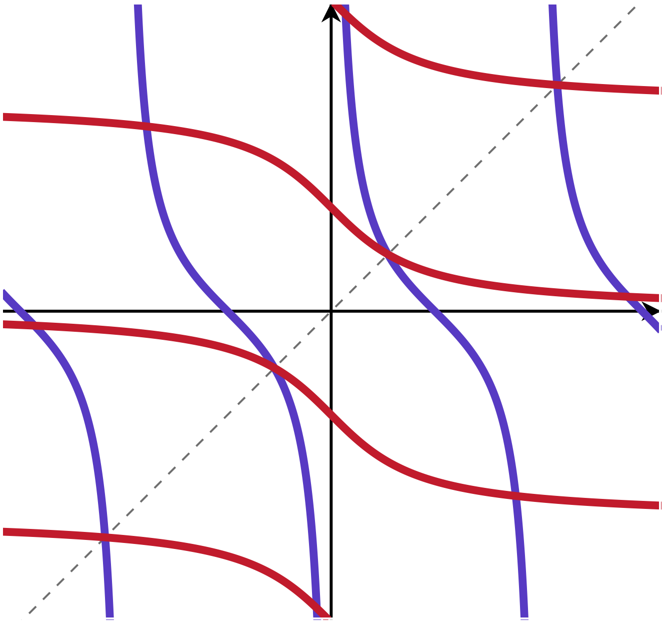
$y = \text{arccsc}(x)$



Domain: $x \leq -1$ and $1 \leq x$ Range: $-\pi/2 \leq y \leq \pi/2$

Domain/range

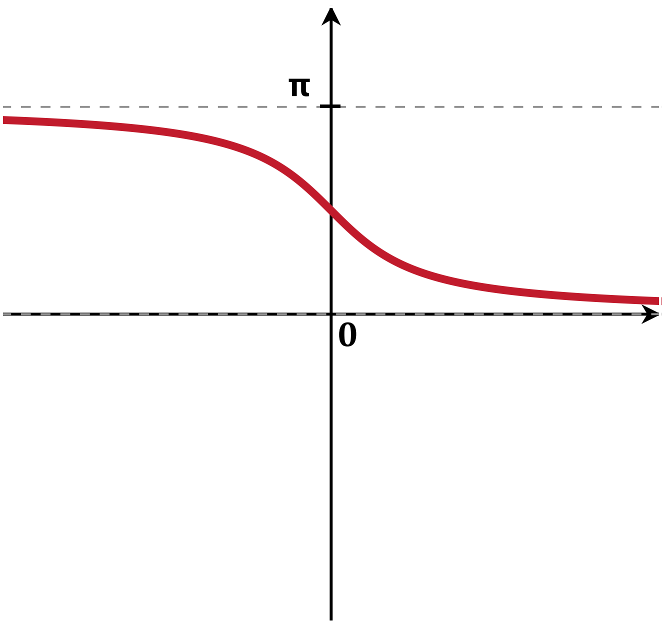
$y = \cot(x)$
$y = \text{arccot}(x)$



Domain: $-\infty \leq x \leq \infty$

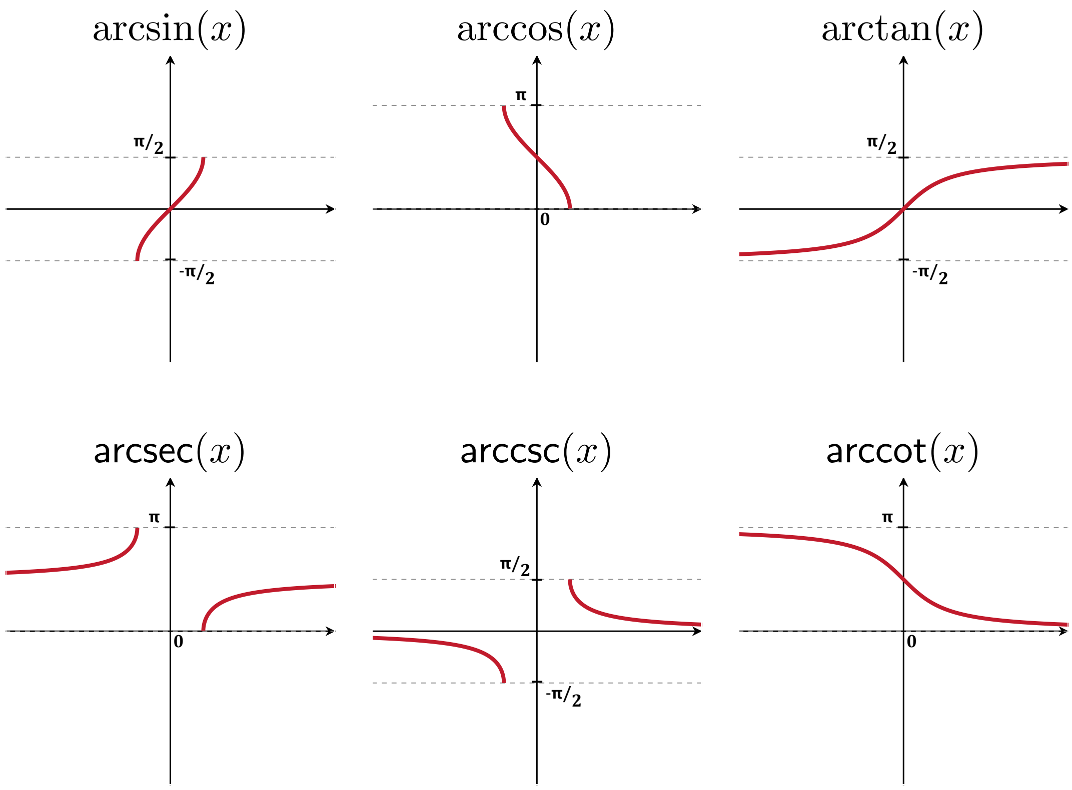
Domain/range

$y = \text{arccot}(x)$



Domain: $-\infty \leq x \leq \infty$ Range: $0 < y < \pi$

Graphs



Using implicit differentiation to calculate $\frac{d}{dx}\arcsin(x)$

If $y = \arcsin(x)$ then $x = \sin(y)$.

Take $\frac{d}{dx}$ of both sides of $x = \sin(y)$:

Left hand side: $$\frac{d}{dx}x = 1$$

Right hand side: $$\frac{d}{dx}\sin(y) = \cos(y) * \frac{dy}{dx} = \cos(\arcsin(x)) * \frac{dy}{dx}$$

So

$$\boxed{\frac{dy}{dx} = \frac{1}{\cos(\arcsin(x))}.}$$

You try:

Use implicit differentiation to calculate the derivatives of

1. $\arccos(x)$
2. $\arctan(x)$

Use the rule

$$\boxed{\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}}$$

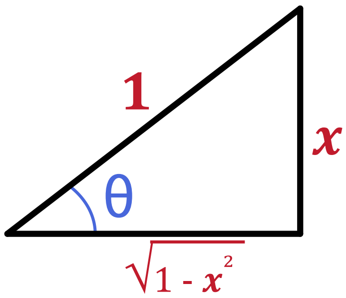
to check your answers, and then to calculate the derivatives of the other inverse trig functions:

1. $\frac{d}{dx}\text{arcsec}(x)$
2. $\frac{d}{dx}\text{arccsc}(x)$
3. $\frac{d}{dx}\text{arccot}(x)$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

$\sin(\theta) = x$



Key: This is a simple triangle to write down whose angle θ has $\sin(\theta) = x$

$$a^2 + x^2 = 1^2 \Rightarrow a = \sqrt{1 - x^2}$$

So $$\cos(\theta) = \cos(\arcsin(x)) = \sqrt{1 - x^2}/1$$

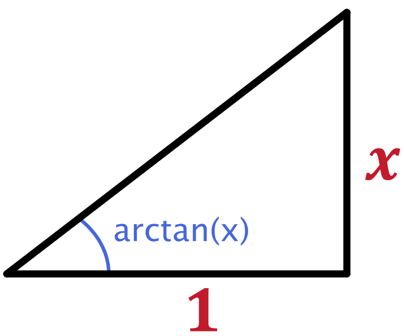
So $$\frac{d}{dx}\arcsin(x) = \frac{1}{\cos(\arcsin(x))} = \boxed{\frac{1}{\sqrt{1 - x^2}}}.$$

Calculating $\frac{d}{dx}\arctan(x)$.

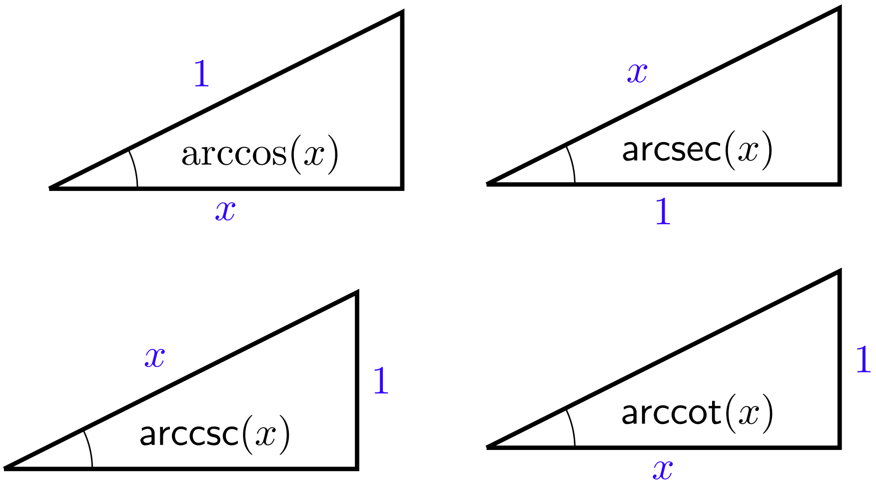
We found that

$$\frac{d}{dx}\arctan(x) = \frac{1}{\sec^2(x)} = \left(\frac{1}{\sec(x)}\right)^2$$

Simplify this expression using



Practice later: To simplify the rest, use the triangles



Answers:

$$\frac{d}{dx}\arcsin(x) = -\frac{d}{dx}\arccos(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan(x) = -\frac{d}{dx}\text{arccot}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\text{arcsec}(x) = -\frac{d}{dx}\text{arccsc}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

You try:

Use the fact that $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$ to calculate the following:

1. $\frac{d}{dx}\arctan(\ln(x))$

2. $\displaystyle\int \frac{1}{1+x^2}dx$

3. $\displaystyle\int \frac{1}{9+x^2}dx$

4. $\displaystyle\int \frac{1}{(1+x)\sqrt{x}}dx$