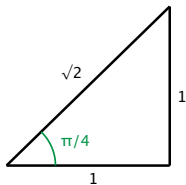
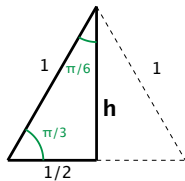


Warm up: Fill out the following tables.

isosceles right triangle:



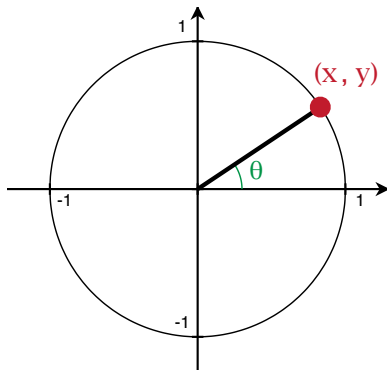
equilateral triangle cut in half:



$$h = \sqrt{1 - (1/2)^2} = \sqrt{3}/2$$

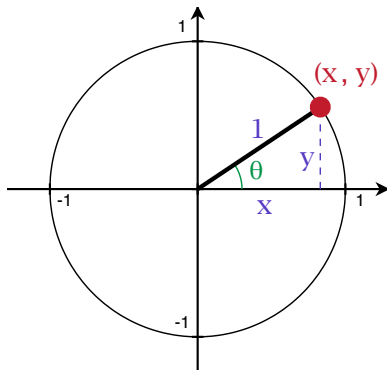
	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\pi/4$						
$\pi/3$						
$\pi/6$						
$f(x)$	$\tan(x)$	$\sec(x)$		$\csc(x)$		$\cot(x)$
$f'(x)$						

Reviewing the unit circle



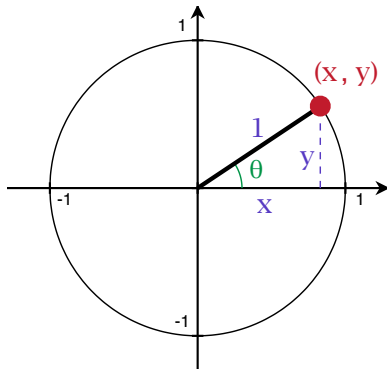
For $0 < \theta < \frac{\pi}{2} \dots$

Reviewing the unit circle



For $0 < \theta < \frac{\pi}{2} \dots$

Reviewing the unit circle



For $0 < \theta < \frac{\pi}{2} \dots$

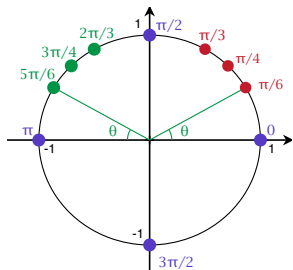
$$\cos(\theta) = \frac{x}{1} = x$$

$$\sin(\theta) = \frac{y}{1} = y$$

Use this idea to extend trig functions to any θ , defining

$$\cos(\theta) = x \quad \sin(\theta) = y.$$

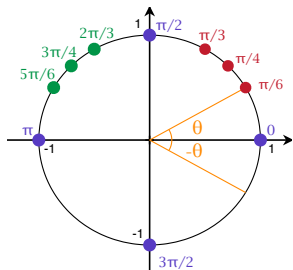
Review: what we can read off of the unit circle



$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$			
$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$			
$\sin(\theta)$	0	1	0	-1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$			
	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$							
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$							

Review: what we can read off of the unit circle



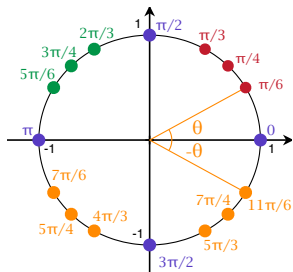
$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta)$$

$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$			
$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$			
$\sin(\theta)$	0	1	0	-1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$			
	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$							
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$							

Review: what we can read off of the unit circle



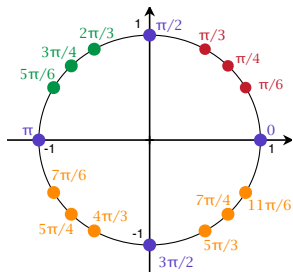
$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta)$$

$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$			
$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$			
$\sin(\theta)$	0	1	0	-1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$			
	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	

Review: what we can read off of the unit circle



$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta)$$

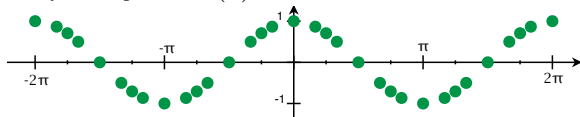
$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$

$$x^2 + y^2 = 1 \implies \cos^2(\theta) + \sin^2(\theta) = 1$$

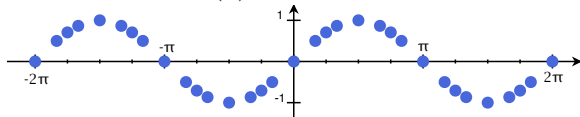
	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$			
$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$			
$\sin(\theta)$	0	1	0	-1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$			
	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	

Plotting on the θ - y axis

Graph of $y = \cos(\theta)$:

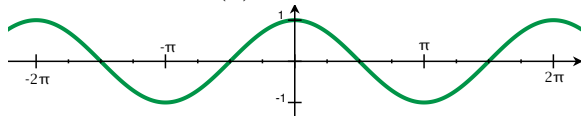


Graph of $y = \sin(\theta)$:

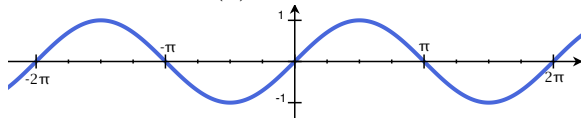


Plotting on the θ - y axis

Graph of $y = \cos(\theta)$:



Graph of $y = \sin(\theta)$:



Trig identities to know and love:

Even/odd:

$$\cos(-\theta) = \cos(\theta) \quad (\text{even}) \quad \sin(-\theta) = -\sin(\theta) \quad (\text{odd})$$

Pythagorean identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

Angle addition:

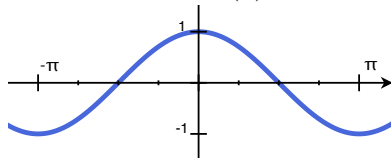
$$\cos(\theta + \phi) = \cos(\theta) \cos(\phi) - \sin(\theta) \sin(\phi)$$

$$\sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \cos(\theta) \sin(\phi)$$

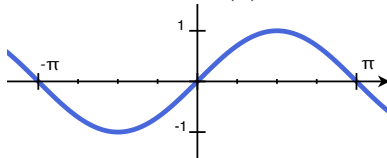
(in particular $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ and $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$)

Other trig functions

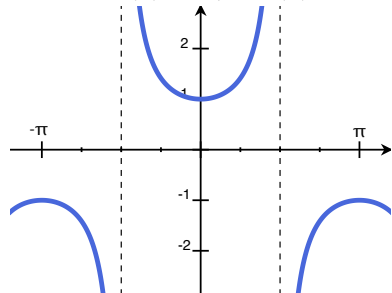
$$y = \cos(\theta)$$



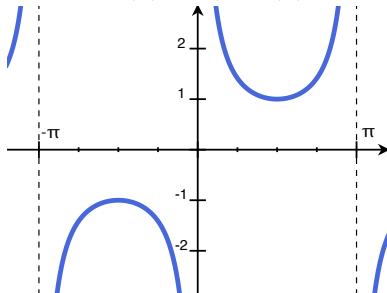
$$y = \sin(\theta)$$



$$\sec(\theta) = 1/\cos(\theta)$$

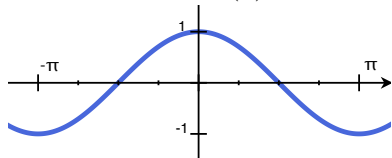


$$\csc(\theta) = 1/\sin(\theta)$$

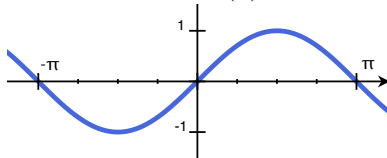


Other trig functions

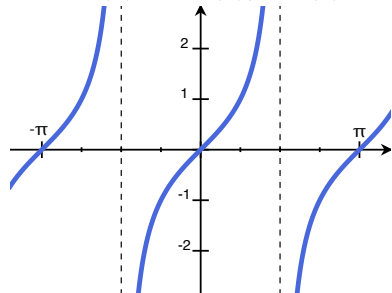
$$y = \cos(\theta)$$



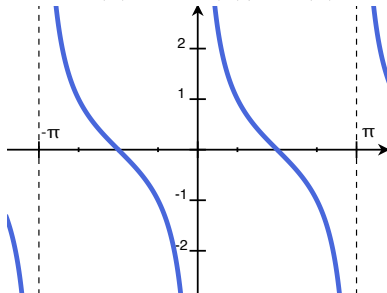
$$y = \sin(\theta)$$



$$\tan(\theta) = \sin(\theta) / \cos(\theta)$$



$$\cot(\theta) = \cos(\theta) / \sin(\theta)$$



Inverse trig functions

Two notations:

$f(x)$	$f^{-1}(x)$
$\sin(x)$	$\sin^{-1}(x) = \arcsin(x)$
$\cos(x)$	$\cos^{-1}(x) = \arccos(x)$
$\tan(x)$	$\tan^{-1}(x) = \arctan(x)$
$\sec(x)$	$\sec^{-1}(x) = \operatorname{arcsec}(x)$
$\csc(x)$	$\csc^{-1}(x) = \operatorname{arccsc}(x)$
$\cot(x)$	$\cot^{-1}(x) = \operatorname{arccot}(x)$

Inverse trig functions

Two notations:

$f(x)$	$f^{-1}(x)$
$\sin(x)$	$\sin^{-1}(x) = \arcsin(x)$
$\cos(x)$	$\cos^{-1}(x) = \arccos(x)$
$\tan(x)$	$\tan^{-1}(x) = \arctan(x)$
$\sec(x)$	$\sec^{-1}(x) = \operatorname{arcsec}(x)$
$\csc(x)$	$\csc^{-1}(x) = \operatorname{arccsc}(x)$
$\cot(x)$	$\cot^{-1}(x) = \operatorname{arccot}(x)$

There are lots of points we know on these functions...

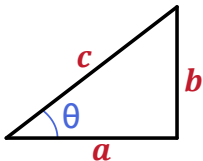
Examples:

1. Since $\sin(\pi/2) = 1$, we have $\arcsin(1) = \pi/2$
2. Since $\cos(\pi/2) = 0$, we have $\arccos(0) = \pi/2$

Etc...

In general:

$\text{arc}___(-)$ takes in a ratio and spits out an angle:



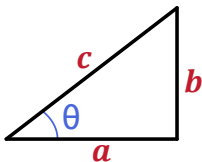
$$\cos(\theta) = a/c \quad \text{so} \quad \arccos(a/c) = \theta$$

$$\sin(\theta) = b/c \quad \text{so} \quad \arcsin(b/c) = \theta$$

$$\tan(\theta) = b/a \quad \text{so} \quad \arctan(b/a) = \theta$$

In general:

$\arcsin(\quad)$ takes in a ratio and spits out an angle:



$$\cos(\theta) = a/c \quad \text{so} \quad \arccos(a/c) = \theta$$

$$\sin(\theta) = b/c \quad \text{so} \quad \arcsin(b/c) = \theta$$

$$\tan(\theta) = b/a \quad \text{so} \quad \arctan(b/a) = \theta$$

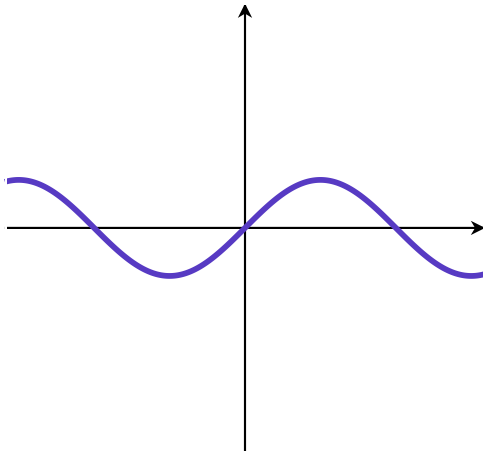
Domain problems:

$$\sin(0) = 0, \quad \sin(\pi) = 0, \quad \sin(2\pi) = 0, \quad \sin(3\pi) = 0, \dots$$

So which is the right answer to $\arcsin(0)$, really?

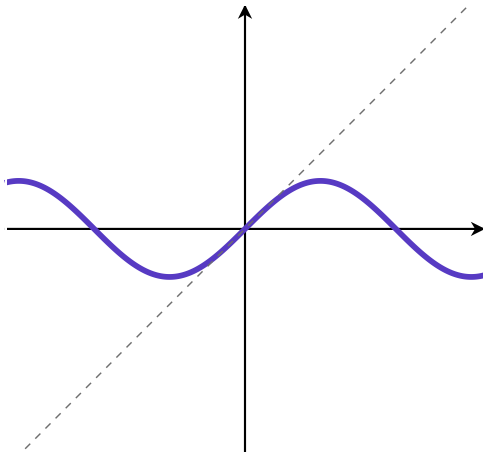
Domain/range

$$y = \sin(x)$$



Domain/range

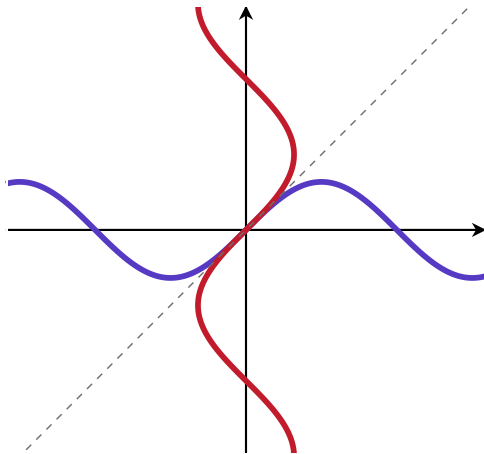
$$y = \sin(x)$$



Domain/range

$$y = \sin(x)$$

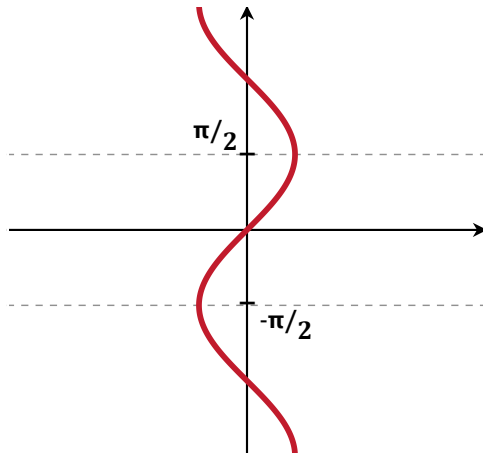
$$y = \arcsin(x)$$



Domain: $-1 \leq x \leq 1$

Domain/range

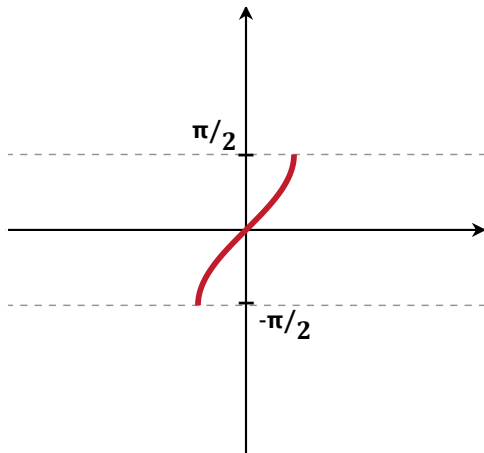
$$y = \arcsin(x)$$



$$\text{Domain: } -1 \leq x \leq 1$$

Domain/range

$$y = \arcsin(x)$$

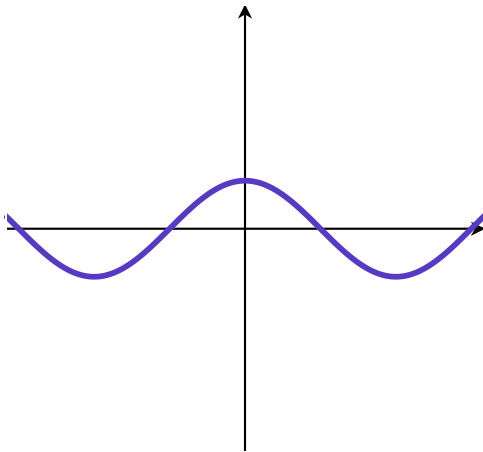


Domain: $-1 \leq x \leq 1$

Range: $-\pi/2 \leq y \leq \pi/2$

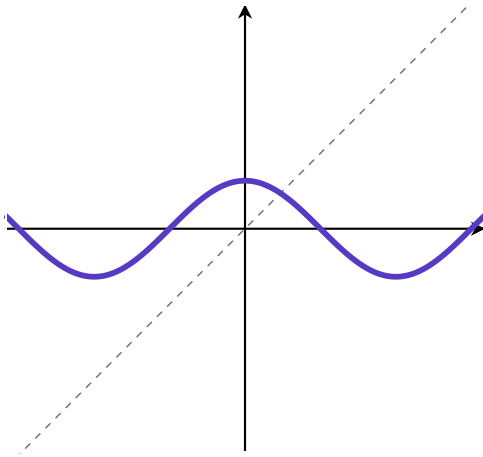
Domain/range

$$y = \cos(x)$$



Domain/range

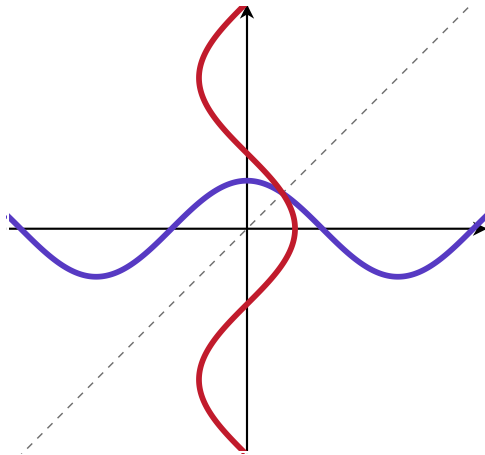
$$y = \cos(x)$$



Domain/range

$$y = \cos(x)$$

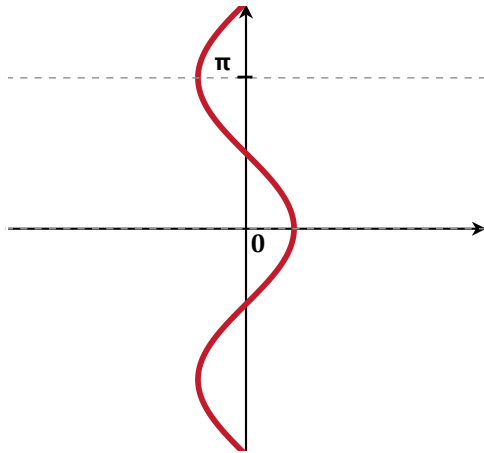
$$y = \arccos(x)$$



Domain: $-1 \leq x \leq 1$

Domain/range

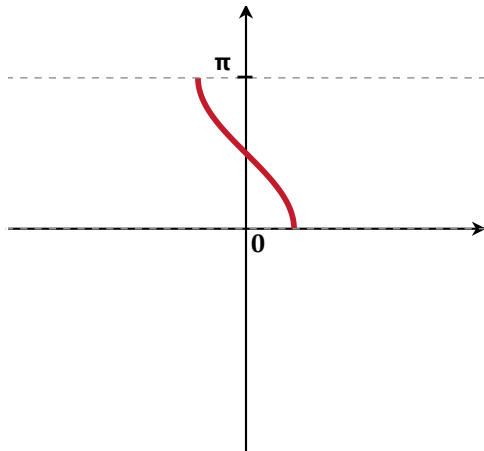
$$y = \arccos(x)$$



$$\text{Domain: } -1 \leq x \leq 1$$

Domain/range

$$y = \arccos(x)$$

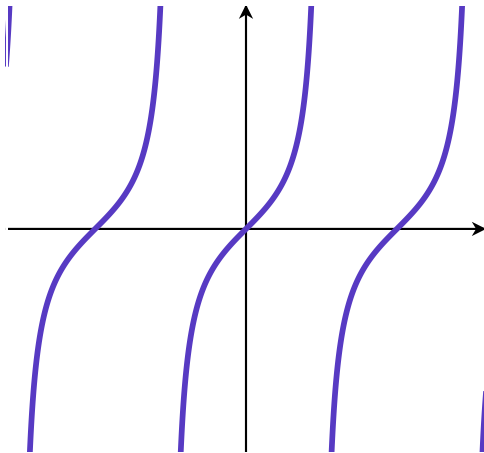


Domain: $-1 \leq x \leq 1$

Range: $0 \leq y \leq \pi$

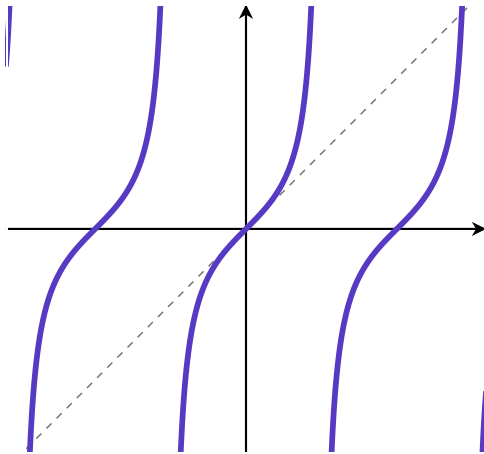
Domain/range

$$y = \tan(x)$$



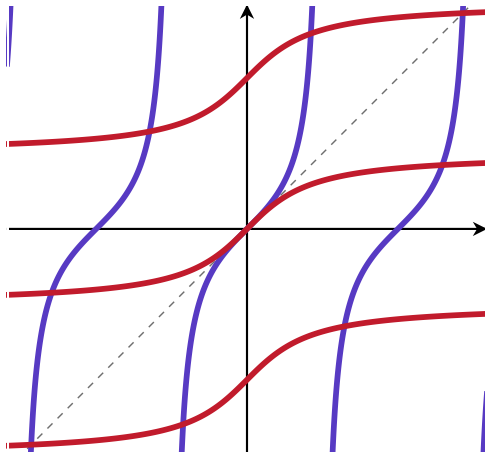
Domain/range

$$y = \tan(x)$$



Domain/range

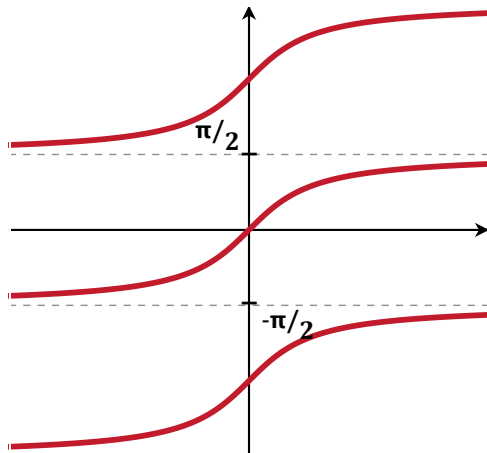
$$y = \tan(x)$$
$$y = \arctan(x)$$



Domain: $-\infty \leq x \leq \infty$

Domain/range

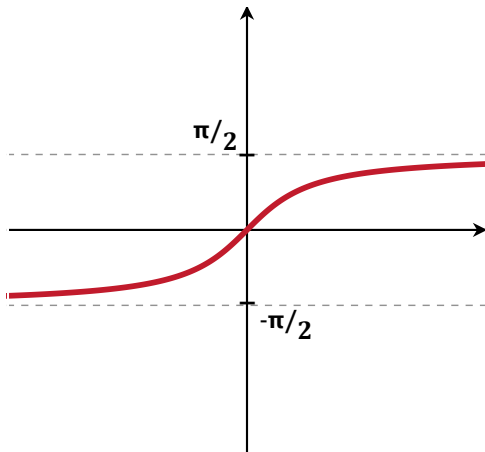
$$y = \arctan(x)$$



Domain: $-\infty \leq x \leq \infty$

Domain/range

$$y = \arctan(x)$$

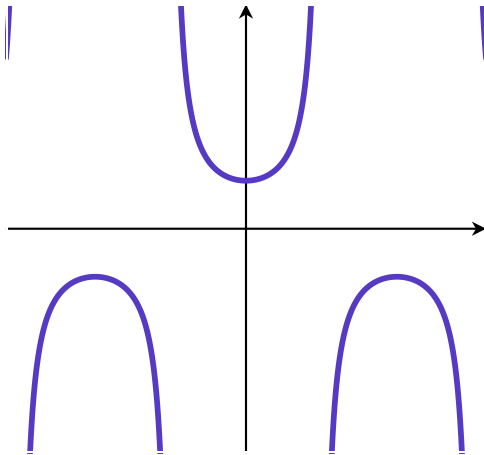


Domain: $-\infty \leq x \leq \infty$

Range: $-\pi/2 < y < \pi/2$

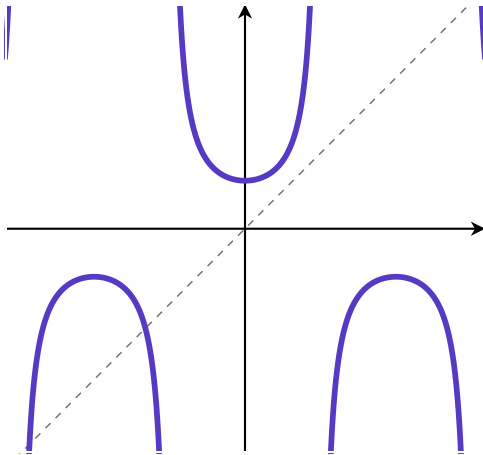
Domain/range

$$y = \sec(x)$$



Domain/range

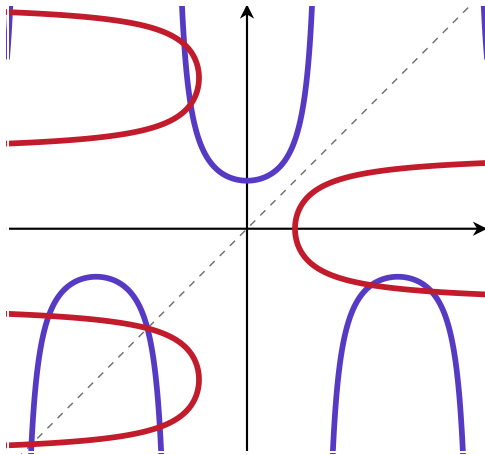
$$y = \sec(x)$$



Domain/range

$$y = \sec(x)$$

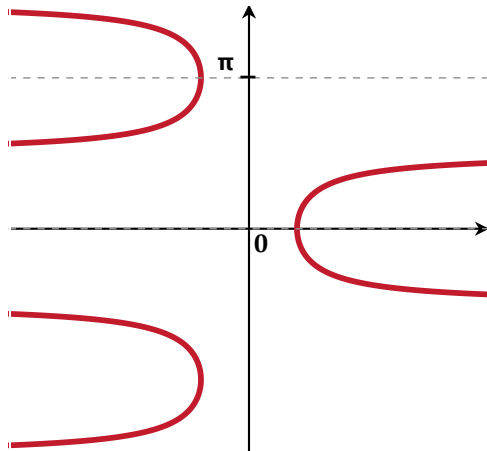
$$y = \operatorname{arcsec}(x)$$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

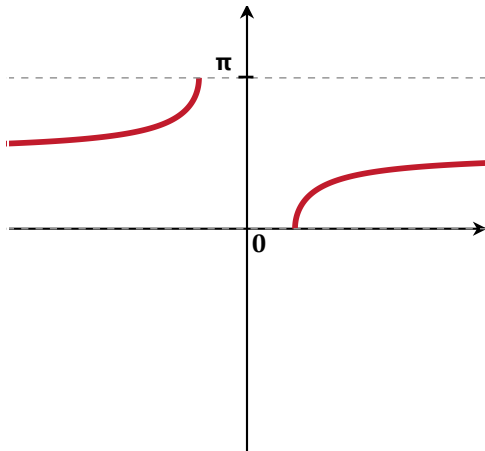
$$y = \operatorname{arcsec}(x)$$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

$$y = \operatorname{arcsec}(x)$$

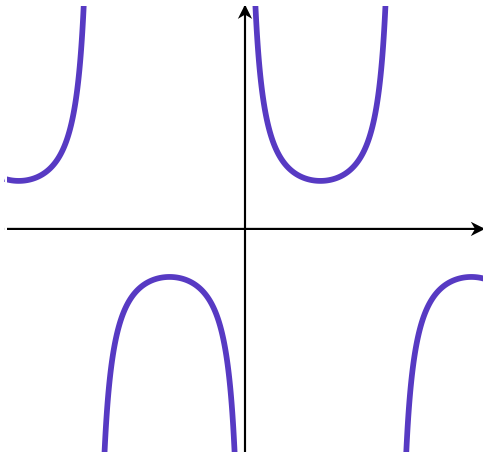


Domain: $x \leq -1$ and $1 \leq x$

Range: $0 \leq y \leq \pi$

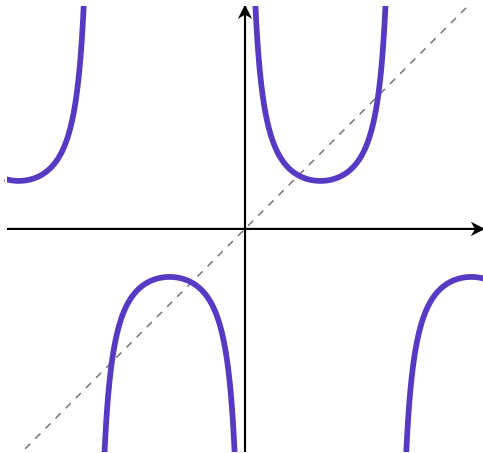
Domain/range

$$y = \csc(x)$$



Domain/range

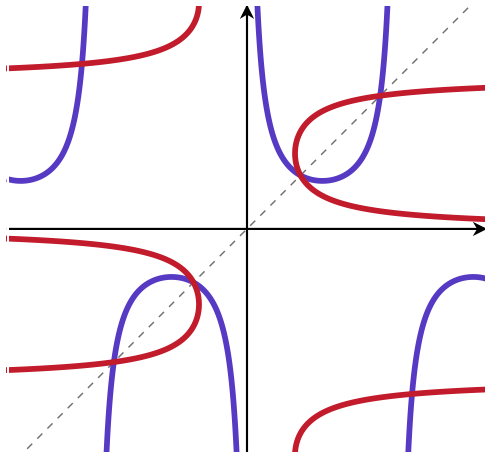
$$y = \csc(x)$$



Domain/range

$$y = \csc(x)$$

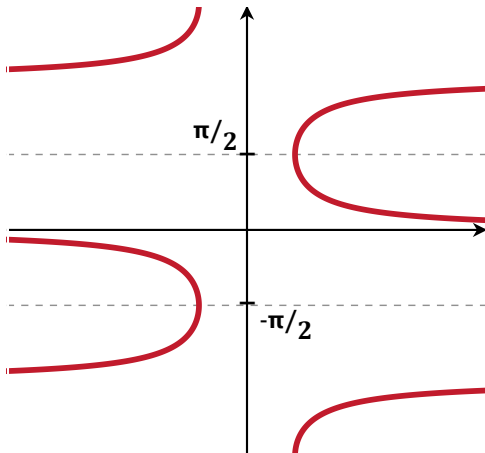
$$y = \operatorname{arccsc}(x)$$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

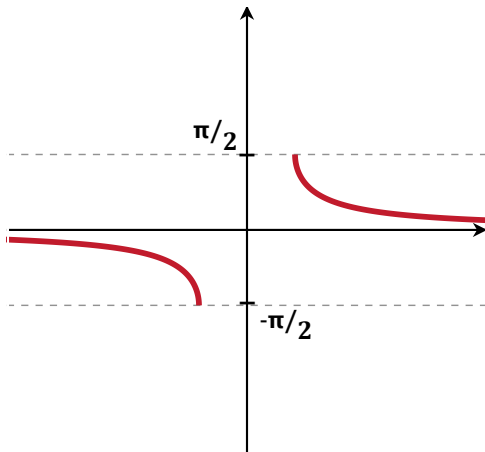
$$y = \operatorname{arccsc}(x)$$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

$$y = \operatorname{arccsc}(x)$$

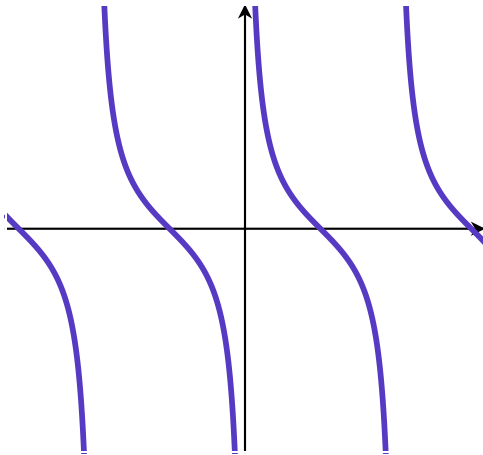


Domain: $x \leq -1$ and $1 \leq x$

Range: $-\pi/2 \leq y \leq \pi/2$

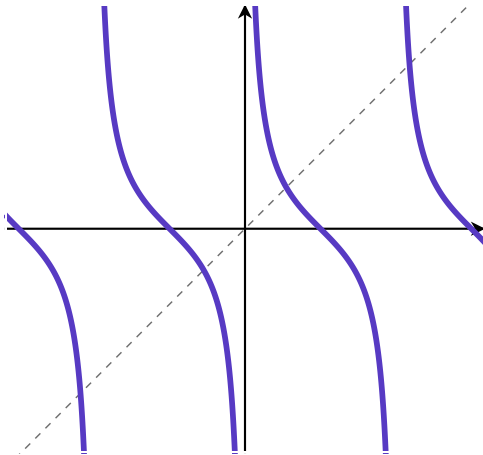
Domain/range

$$y = \cot(x)$$



Domain/range

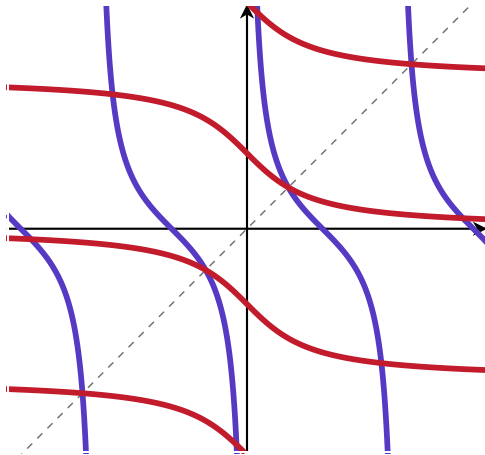
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Domain/range

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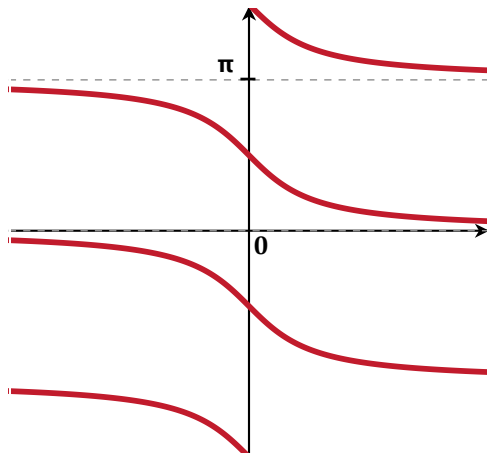
$$y = \operatorname{arccot}(x)$$



Domain: $-\infty \leq x \leq \infty$

Domain/range

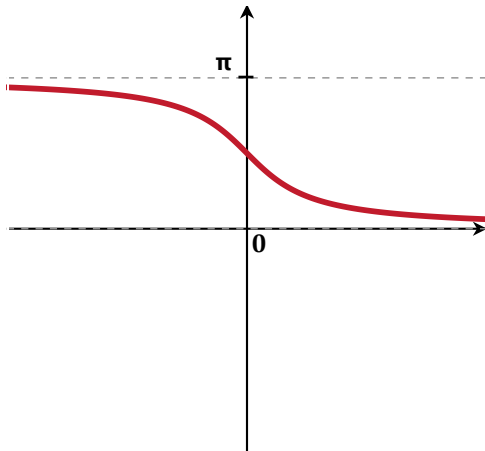
$$y = \operatorname{arccot}(x)$$



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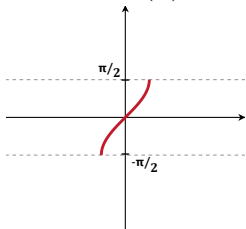


Domain: $-\infty \leq x \leq \infty$

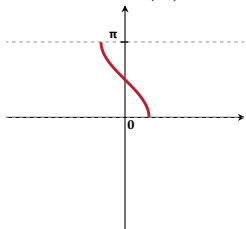
Range: $0 < y < \pi$

Graphs

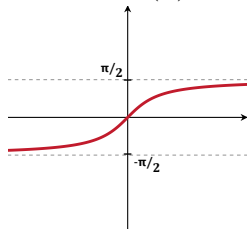
$\arcsin(x)$



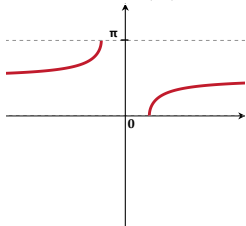
$\arccos(x)$



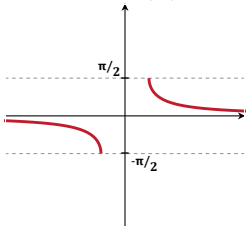
$\arctan(x)$



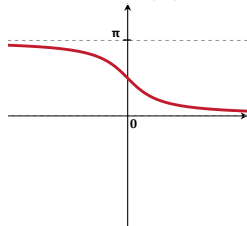
$\operatorname{arcsec}(x)$



$\operatorname{arccsc}(x)$



$\operatorname{arccot}(x)$



Using implicit differentiation to calculate $\frac{d}{dx} \arcsin(x)$

If $y = \arcsin(x)$ then $x = \sin(y)$.

Using implicit differentiation to calculate $\frac{d}{dx} \arcsin(x)$

$$\boxed{\text{If } y = \arcsin(x) \text{ then } x = \sin(y).}$$

Take $\frac{d}{dx}$ of both sides of $x = \sin(y)$:

Using implicit differentiation to calculate $\frac{d}{dx} \arcsin(x)$

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Take $\frac{d}{dx}$ of both sides of $x = \sin(y)$:

$$\text{Left hand side: } \frac{d}{dx} x = 1$$

$$\text{Right hand side: } \frac{d}{dx} \sin(y) = \cos(y) * \frac{dy}{dx} = \cos(\arcsin(x)) * \frac{dy}{dx}$$

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So

$$\boxed{\frac{dy}{dx} = \frac{1}{\cos(\arcsin(x))}.}$$

You try:

Use implicit differentiation to calculate the derivatives of

1. $\arccos(x)$
2. $\arctan(x)$

Use the rule

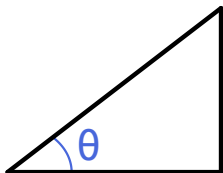
$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

to check your answers, and then to calculate the derivatives of the other inverse trig functions:

1. $\frac{d}{dx} \operatorname{arcsec}(x)$
2. $\frac{d}{dx} \operatorname{arccsc}(x)$
3. $\frac{d}{dx} \operatorname{arccot}(x)$

Simplifying $\cos(\arcsin(x))$

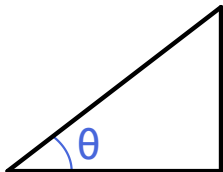
Call $\arcsin(x) = \theta$.



Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

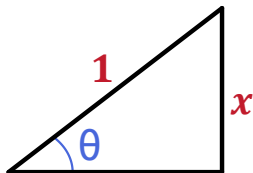
$$\sin(\theta) = x$$



Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

$$\sin(\theta) = x$$

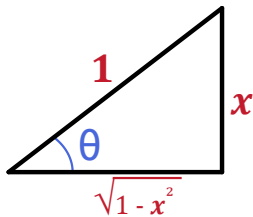


Key: This is a simple triangle to write down
whose angle θ has $\sin(\theta) = x$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

$$\sin(\theta) = x$$



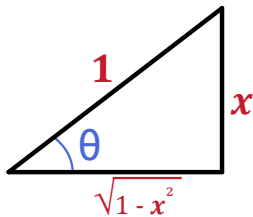
Key: This is a simple triangle to write down
whose angle θ has $\sin(\theta) = x$

$$a^2 + x^2 = 1^2 \Rightarrow a = \sqrt{1-x^2}$$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

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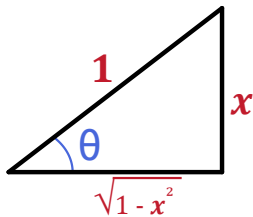
$$a^2 + x^2 = 1^2 \Rightarrow a = \sqrt{1-x^2}$$

So $\cos(\theta) = \sqrt{1-x^2}/1$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

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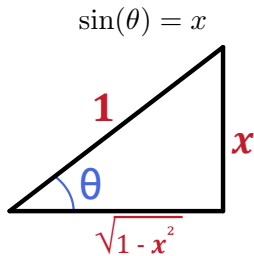
Key: This is a simple triangle to write down
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$$a^2 + x^2 = 1^2 \Rightarrow a = \sqrt{1-x^2}$$

So $\cos(\theta) = \cos(\arcsin(x)) = \sqrt{1-x^2}/1$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.



Key: This is a simple triangle to write down
whose angle θ has $\sin(\theta) = x$

$$a^2 + x^2 = 1^2 \Rightarrow a = \sqrt{1-x^2}$$

So $\cos(\theta) = \cos(\arcsin(x)) = \sqrt{1-x^2}/1$

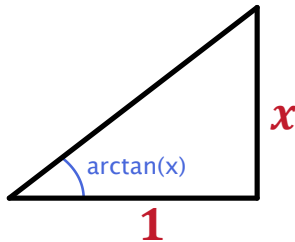
So
$$\frac{d}{dx} \arcsin(x) = \frac{1}{\cos(\arcsin(x))} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

Calculating $\frac{d}{dx} \arctan(x)$.

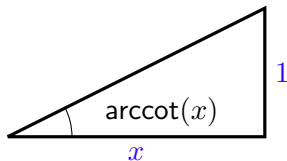
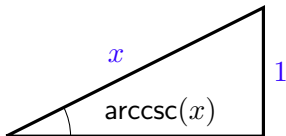
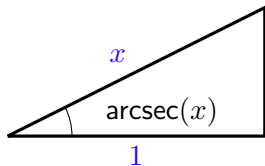
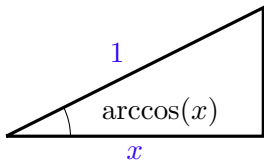
We found that

$$\frac{d}{dx} \arctan(x) = \frac{1}{\sec^2(x)} = \left(\frac{1}{\sec(x)} \right)^2$$

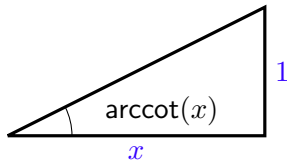
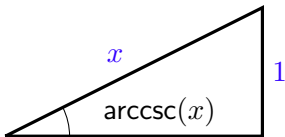
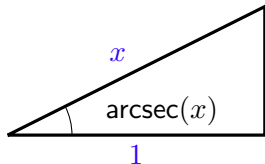
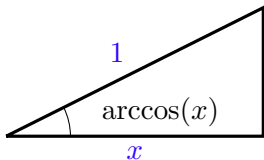
Simplify this expression using



Practice later: To simplify the rest, use the triangles



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Answers:

$$\frac{d}{dx} \arcsin(x) = -\frac{d}{dx} \arccos(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arctan(x) = -\frac{d}{dx} \operatorname{arccot}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \operatorname{arcsec}(x) = -\frac{d}{dx} \operatorname{arccsc}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

You try:

Use the fact that $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$ to calculate the following:

1. $\frac{d}{dx} \arctan(\ln(x))$

2. $\int \frac{1}{1+x^2} dx$

3. $\int \frac{1}{9+x^2} dx$

4. $\int \frac{1}{(1+x)\sqrt{x}} dx$