Welcome to Calculus 2!

Logistics:

Professor Zajj Daugherty, NAC 6-301
Office hours: Monday 2:30–3:30, Wednesday 11–12.
For now, the section's website is at

http://math.sci.ccny.cuny.edu/pages?name=m202FGs16

Grades: You grades will be based on

1. 15%: Homework and occasional quizzes:
   Homework though WebAssign (www.webassign.net)
   Course key: ccny 4222 6935
   Online homework due 1 week after content is covered.
2. 45%: Midterms
   Midterms will be in class, and are tentatively scheduled for Wednesday 3/16 and Wednesday 4/20.
3. 40%: Course-wide final
   The final will be modeled after the homework list on the course-wide syllabus. Most, but not all, will appear on your WebAssign homeworks.

This course will cover...

- Chapter 5: Inverse functions (exponential, logarithmic, inverse trig, and hyperbolic functions; indeterminate forms and L'Hospital's rule)
- Chapter 6: Techniques of integration (calculating integrals algebraically, educated guessing); skip: 6.4.
- Chapter 7: Applications of integration (area, volume, arc length, physics); skip: 7.5, 7.7.
- Chapter 9: Parametric equations and polar coordinates; skip 9.5.
- Other stuff: Conic sections

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Note that $\sqrt{x}$ is only a function when we go to extra effort to decide that we're always going to choose the positive answer.

3. Let bacteria grow, and measure population over time.
Consider $N : \mathbb{N} \to \mathbb{N}$ by $N(t) = \#$ bacteria at time t.

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Answer: between 4 and 5 hours

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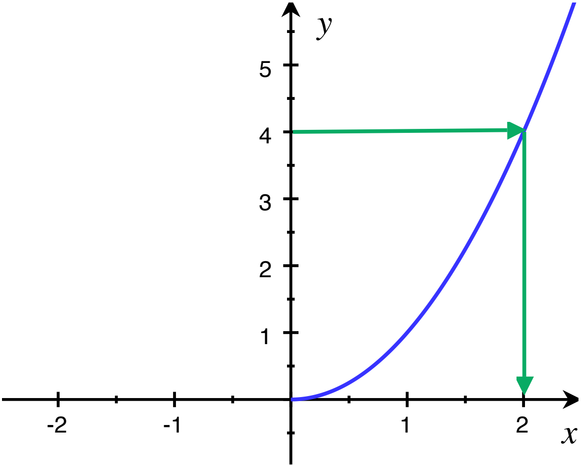

If $y = x^2$ and $x \geq 0$, then $x = |\sqrt{y}|$.

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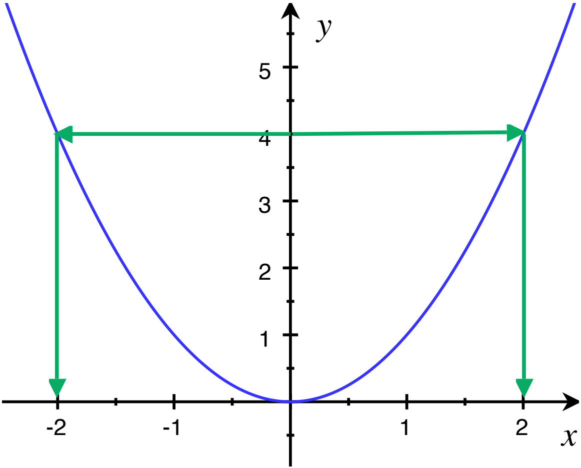
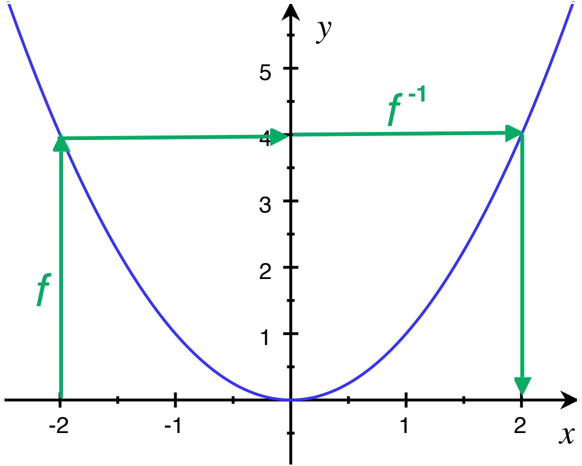


If $y = x^2$, then $x = |\sqrt{y}|$ or $-|\sqrt{y}|$. Which one???

If $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is given by

$$f(x) = x^2, \qquad \text{then} \qquad |\sqrt{x}| \text{ is not the inverse.}$$



If $y = x^2$ and $x < 0$, then $x \neq |\sqrt{y}|$!

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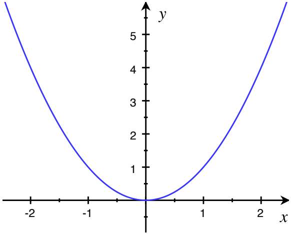

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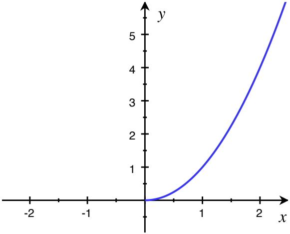

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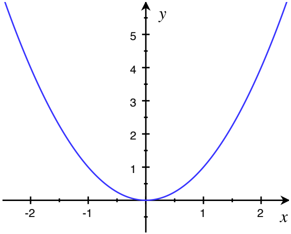

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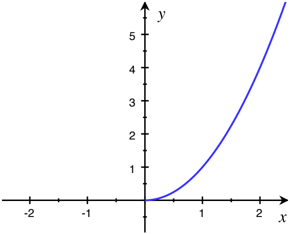

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Horizontal line test: A function is one-to-one if and only if no horizontal line intersects the function's graph more than once.

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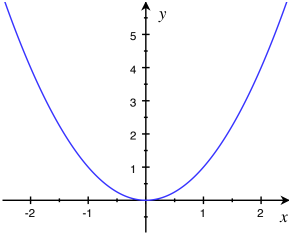

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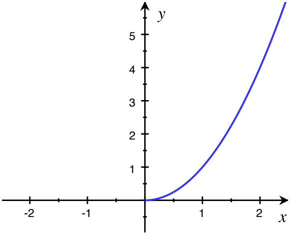

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Answer: A function is invertible if and only if it is one-to-one.

Graphing inverses

For a one-to-one function f, we have

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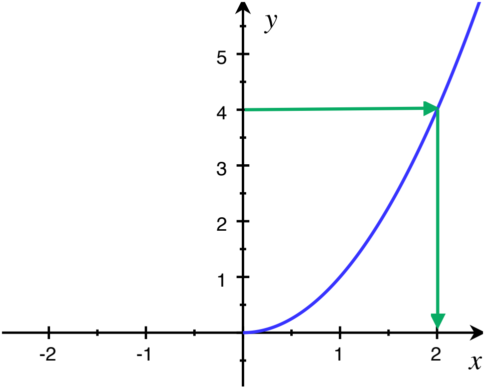

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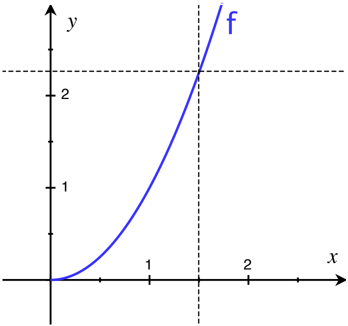

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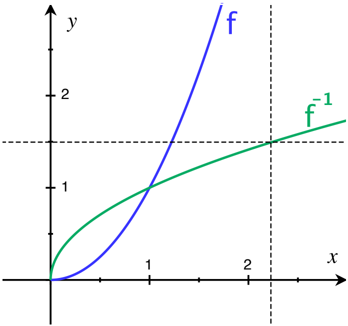

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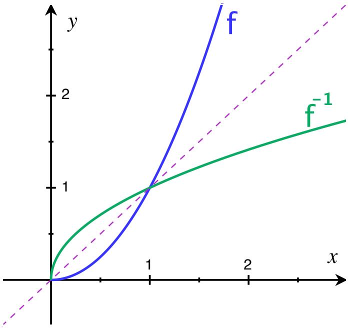

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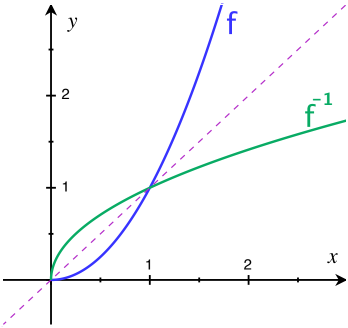

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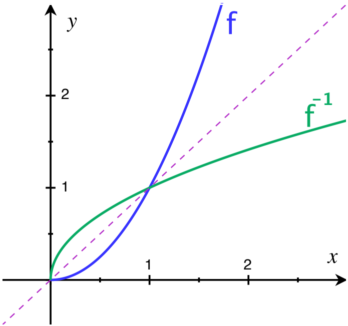

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the domain of f is the range of f^{-1},

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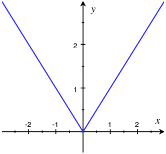
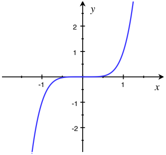
You try:

- For each of the following functions, (a) give the domain and range of f, and (b) decide if f is invertible.
- If f is invertible, then (c) sketch a graph of f^{-1}, (d) give the domain and range of f^{-1}, and (e) try to write a formula for f^{-1}.
- If f is not invertible over all of the real numbers, what is a restricted domain over which f is invertible? Over that restricted domain, do (c) and (d) from above.

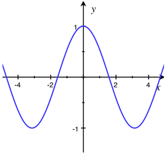
(1) $f(x) = |x|$



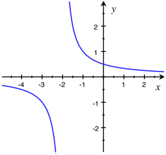
(2) $f(x) = x^5$



(3) $f(x) = \cos(x)$



(4) $f(x) = 1/(x+2)$



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Note: As the book outlines, you can alternatively start with $f(x) = y$, solve for x, and then swap x and y at the end. You will get the same answer either way.

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You try:

1. Check that if $f(x) = x^3 + 2$ then $f^{-1}(x) = (x-2)^{1/3}$ by calculating $f(f^{-1}(x))$ and $f^{-1}(f(x))$.
2. For the following functions, calculate $f^{-1}(x)$ and verify your answer as above. (a) $f(x) = 3/(x-1)$ (b) $f(x) = 5\sqrt{x-2}$

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You try:

For each of the following:

(a) Show that f is one-to-one.

(Show $f' \geq 0$ over the domain, and that f is never constant)

(b) Use the formula $\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$ to calculate $\frac{d}{dx} f^{-1}(x)\Big|_{x=8}$.

(c) Calculate $f^{-1}(x)$, and state the domain and range of $f^{-1}(x)$.

(d) Calculate $\frac{d}{dx} f^{-1}(x)\Big|_{x=8}$ directly using the formula from part (c). Check it against your answer for part (b).

(e) Sketch graphs of $f(x)$ and $f^{-1}(x)$ on the same axis. Then sketch a tangent line to $f^{-1}(x)$ at $x = 8$ and visually check that your answer to parts (b) and (d) roughly match the slope of this line.

(1) $f(x) = x^3$.	(2) $f(x) = 9 - x^2, 0 \leq x \leq 3$.