Name: Solutions

Math 201 - Quiz 4 - Thursday, Nov 29, 2018

Instructions: Show your work, justify your answers, and write clearly.

Recall the following formulas:
Volume of a cylinder with height h and radius r: $V = \pi r^2 h$
Area of the vertical sides of a cylinder with height h and radius r: $A = 2\pi r h$
Area of a circle of radius r: $A = \pi r^2$.

1. Suppose you want to design a can that holds 230 cm^3 (cubic centimeters).
   (a) In terms of the radius r and the height h, what is the total surface area S of the can?

   $S =$ top + bottom + sides $= 2(\pi r^2) + 2\pi r h$

   (b) Given that the volume is 230 cm^3, what is the surface area S of the can **just in terms of the radius** r?

   We have $230 = \pi r^2 h$, so $h = 230/\pi r^2$.

   So $S = 2\pi r^2 + 2\pi r\left(\frac{230}{\pi r^2}\right)$

   (c) What is the range for r? (What possible values can r take?)

   $r \geq 0$

   (d) Briefly explain how to finish computing the minimum surface area of a can of volume 230 cm^3.

   ① Take $\frac{d}{dr}$ of S (from (b)), and compute the roots (solve $\frac{dS}{dr} = 0$ for r).

   ② Plug in the r-values from ①, as well as from endpoints $(r=0)$. Since r doesn't have an upper bound, compute $\lim_{r \to \infty} S$.

   ③ The minimum S is the smallest value from ②.

2. Compute $\int x^2 + 5 + \sin(x)\,dx$.

$$= \frac{x^3}{3} + 5x - \cos(x) + C$$

Check: $\frac{d}{dx}\left(\frac{x^3}{3} + 5x - \cos(x) + c\right)$

$$= \frac{\cancel{3}x^2}{\cancel{3}} + 5 - (-\sin(x)) + 0 \quad \checkmark$$

3. Compute $\lim_{x\to\infty} x^{1/x}$.

Let $L = \lim_{x\to\infty} x^{1/x}$.

Then $\ln(L) = \ln\left(\lim_{x\to\infty} x^{1/x}\right)$

$$= \lim_{x\to\infty} \ln(x^{1/x})$$

$$= \lim_{x\to\infty} \underbrace{\frac{1}{x}\ln(x)}_{= \frac{\ln(x)}{x}}$$

$\ln(x) \to \infty$
$x \to \infty$
: use L'Hospital's rule

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}(\ln(x))}{\frac{d}{dx}x}$$

$$= \lim_{x\to\infty} (1/x)/1 = \lim_{x\to\infty} \frac{1}{x} = \underline{0}$$

So $\lim_{x\to\infty} x^{1/x} = e^{\ln(L)} = e^0 = \boxed{1}$