Name: Solutions

Math 201 - Quiz 2 - Thursday, Sept 27, 2018

Instructions: Unless otherwise stated, show your work, justify your answers, and write clearly. Put numerical answers in indicated boxes. If a limit is undefined, you may write "und".

1. For each of the following, give the limit value. No need to justify.

$$\lim_{x\to 0}\frac{\sin(x)}{x} = \boxed{1} \qquad \lim_{x\to\infty}\frac{\sin(x)}{x} = \boxed{0} \qquad \lim_{x\to\pi/2}\frac{\sin(x)}{x} = \boxed{\frac{2}{\pi}}$$

$\sin(\pi/2) = 1$

2. Consider $f(x) = \dfrac{5}{3+e^{2x}}$.
   (a) Compute the domain and range of $f(x)$.

Domain: $\boxed{\text{all real } x \quad (-\infty, \infty)}$

Range: $\boxed{(0, 5/3)}$

The range of... e^{2x} : $(0, \infty)$

$3+e^{2x}$: $(3, \infty)$

$\dfrac{1}{3+e^{2x}}$: $(0, \frac{1}{3})$

(b) Compute the following limits.

$$\lim_{x\to-\infty}\frac{5}{3+e^{2x}} = \boxed{\frac{5}{3}} \qquad \lim_{x\to 0}\frac{5}{3+e^{2x}} = \boxed{\frac{5}{4}} \qquad \lim_{x\to\infty}\frac{5}{3+e^{2x}} = \boxed{0}$$

as $x \to -\infty$, $e^{2x} \to 0$

plug in: $\dfrac{5}{3+e^0} = \dfrac{5}{3+1}$

as $x \to \infty$, $3+e^{2x} \to \infty$.

3. Compute the average rate of change for the function $f(x) = x^2$ over the interval $[1, 5]$. (Write intelligibly while justifying your answer!)

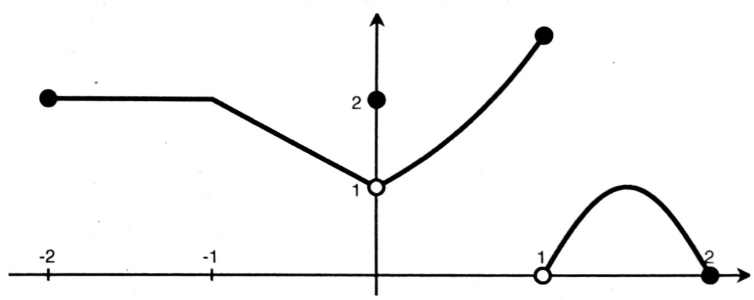
$$\text{avg rate of change over } [1,5] = \frac{f(5) - f(1)}{5 - 1}$$

$$= \frac{5^2 - 1^2}{4}$$

$$= \boxed{\frac{25-1}{4}} = 6$$

4. Consider the piecewise function defined over the interval $[-2, 2]$ given by

$$f(x) = \begin{cases} 2 & -2 \le x \le -1 \\ -x+1 & -1 < x < 0 \\ 2 & x = 0 \\ e^x & 0 < x \le 1 \\ \sin(\pi x) & 1 < x \le 2 \end{cases}$$



Compute the following limits. No need to show work (you may use the graph).

$\lim_{x \to 0^-} f(x) = \boxed{1}$ $\quad \lim_{x \to 0^+} f(x) = \boxed{1}$

$\lim_{x \to 1^-} f(x) = \boxed{e}$ $\quad \lim_{x \to 1^+} f(x) = \boxed{0}$

5. For which values a in the interval $[-2, 2]$ does the (two-sided) limit $\lim_{x \to a} f(x)$ **not** exist?

Answer(s): $\boxed{-2, 1, 2}$

Justification:

$a = -2$ and 2 are end points, so only have one-sided limits. At $a = 1$, the left limit is not equal to the right limit.