1. Local extrema of

$$f(x) = x^4 - 62x^2 + 120x + 9$$

$$f'(x) = 4x^3 - 124x + 120$$

$$= 4(x^3 - 31x + 30) = 4(x-1)\underbrace{(x^2 + x - 30)}$$

Factoring: $f'(1) = 0$:

$$\begin{array}{r|l} & x^2 + x - 30 \\ \hline x - 1 & x^3 - 31x + 30 \\ & -(x^3 - x^2) \\ \hline & x^2 - 31x + 30 \\ & -(x^2 - x) \\ \hline & -30x + 30 \end{array}$$

$$= \boxed{4(x-1)(x+6)(x-5)}$$

$$\begin{array}{ccccccccc} f' & - & 0 & + & 0 & - & 0 & + \\ & & -6 & & 1 & & 5 & \\ & \searrow & & \nearrow & & \searrow & & \nearrow \end{array}$$

local mins : $\boxed{x = -6, 5}$

local max's : $\boxed{x = 1}$

2. Local extrema of

$$f(x) = (x-1)(x+2)^2.$$

To make factoring easier, I don't expand first...

$$f' = (x+2)^2 + 2(x-1)(x+2)$$

$$= (x+2)(x+2+2x-2)$$

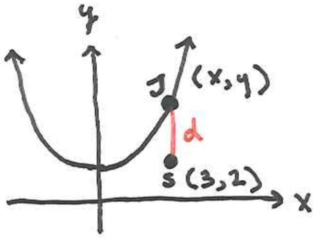
$$= (x+2)(3x)$$

$$\begin{array}{cccccc} f' & + & & - & & + \\ & & -2 & & 0 & \\ & \nearrow & & \searrow & & \nearrow \end{array}$$

local max : $\boxed{x = -2}$

local min : $\boxed{x = 0}$

9. An enemy jet is flying along the curve $y = x^2 + 2$. A soldier is placed at the point $(3,2)$. At what point will the jet be when it's closest to the soldier?



optimize: $d = \sqrt{(x-3)^2 + (y-2)^2}$

constraint: $y = x^2 + 2$

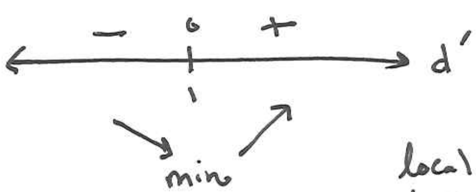
plug in

$$d(x) = \sqrt{(x-3)^2 + \underbrace{(x^2+2-2)^2}_{x^4}} = \sqrt{x^4 + x^2 - 6x + 9}$$

$$\text{so } d'(x) = \frac{\overset{2}{\cancel{4}}x^3 + \cancel{2}x - \overset{3}{\cancel{6}}}{\cancel{2}\sqrt{x^4+x^2-6x+9}} = \frac{2x^3 + x - 6}{\sqrt{x^4 + (x-3)^2}}$$

denom: $x^4 + (x-3)^2$ always positive

num: $2x^3 + x - 3 = 0$ @ $x = 1$ (only)



local min is $\underline{\underline{\text{only}}}$ turn → absolute min.

⇒ $\boxed{(1,3)}$

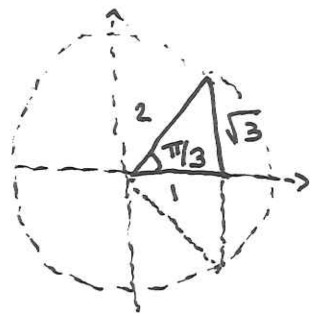
10. Find local extrema of

$f = -x + 2\sin(x)$ in $[0, 2\pi]$.

$f' = -1 + 2\cos(x)$

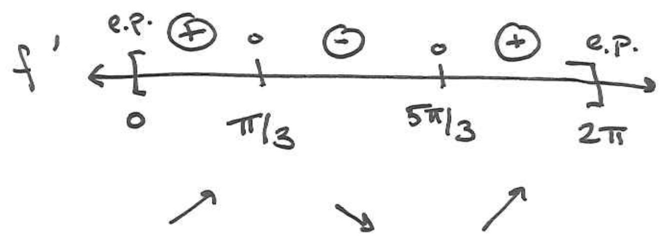
find crit pts: $-1 + 2\cos(x) = 0$

so $\cos(x) = 1/2$



In the interval $[0, 2\pi]$:

$X = \pi/3,\ \underbrace{2\pi - \frac{\pi}{3}}_{5\pi/3}$



local mins: $\boxed{x = 0,\ 5\pi/3}$

local max's: $\boxed{x = \pi/3,\ 2\pi}$

11. Divide 15 into two parts so that the square of one times the cube of the other is biggest.

Constraint: $x+y=15$, $15 \geq x, y \geq 0$.

Maximize: $P = x^2 \cdot y^3$.

$x+y=15 \rightarrow y = 15-x$ or $\underline{x = 15-y}$.

(since square is smaller, or whatever)

so

$$P = (15-y)^2 \cdot y^3$$

$$\frac{d}{dy}P = -2(15-y)y^3 + 3(15-y)^2 y^2$$

$$= (15-y)\underbrace{\left(-2y^3 + 3(15-y)y^2\right)}_{45y^2 - (2+3)y^3}$$

$$= 5(15-y)y^2(9-y)$$

y	P
0	0
9	$\boxed{6^2 \cdot 9^3}$
15	0

$\rightarrow$ max P @ $x = 6, y = 9$

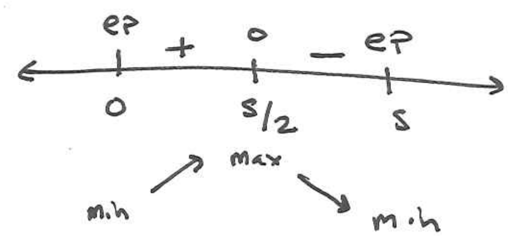
12. Suppose the sum of two numbers is fixed. Show their product is maximized when one is half the total sum.

Constraint: $x+y = S$ $S \geq x, y \geq 0$

maximize $P = xy$

$$= (S-y)y = Sy - y^2$$

$$\frac{dP}{dy} = S - 2y = 0 \quad \text{when} \quad y = \frac{S}{2}$$



-or-

y	P
0	0
$S/2$	$S^2/4$
S	0

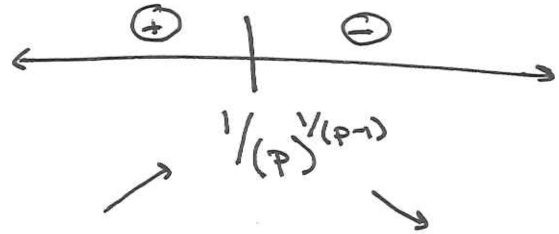
$\leftarrow$ max.

14. Which fraction exceeds its p^{th} power by the maximum amount?

maximize: $f(x) = x - x^p$ (assume $p > 0$)

$f' = 1 - px^{p-1} = 0$ if $x^{p-1} = \frac{1}{p}$

$x = \frac{1}{(p)^{1/(p-1)}}$

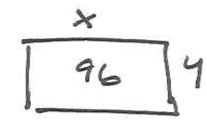


$$\boxed{x = \frac{1}{p^{1/p-1}}}$$

15. Find the dimensions of the rectangle of area 96 cm² which has min perimeter. What is that perimeter?

constraint: $xy = 96$, $x, y \geq 0$.

optimize: $2x + 2y = P$



$xy = 96 \rightarrow y = 96 \cdot \frac{1}{x}$

so

$P = 2\left(x + 96 \cdot \frac{1}{x}\right)$

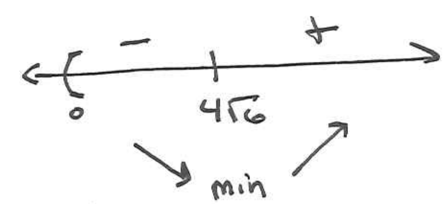
so $P' = 2\left(1 - 96 \cdot \frac{1}{x^2}\right) = 0$

when $x^2 = 96$

$x = \pm 4\sqrt{6}$

↑ pos since $x \geq 0$.

P'



only turn around ⇒ absolute min.

$$\boxed{\text{dimensions: } 4\sqrt{6} \times 4\sqrt{6}, \quad \text{perim: } 16\sqrt{6}}$$