L'Hospital's rule: if f and g are differentiable, $g'(x) \neq 0$ near a (but $g'(a) = 0$ is ok), and

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \text{ or } \lim_{x \to a} f(x) = \lim_{x \to a} g(x) = \pm\infty,$$

then

$$\lim_{x \to a} f(x)/g(x) = \lim_{x \to a} f'(x)/g'(x).$$

Same goes for one-sided limits and $x \to \pm\infty$.

You try: Use whatever methods you have at your disposal to calculate the following limits.

$$\text{(1)} \lim_{x \to 0^+} \frac{\sin(x)}{1 - \cos(x)}, \quad \text{(2)} \lim_{x \to 0} \frac{3^x - e^x}{x}, \quad \text{(3)} \lim_{x \to 0} \frac{2x^2 + x}{3x^2 + 1},$$

$$\text{(4)} \lim_{x \to \infty} x^{-\ln(x)}, \quad \text{(5)} \lim_{x \to \infty} \tan^{-1}(e^x/x), \quad \text{(6)} \lim_{x \to -\infty} \tan^{-1}(e^x/x),$$

$$\text{(7)} \lim_{x \to \infty} e^{-x} \tan^{-1}(x), \quad \text{(8)} \lim_{x \to 0^-} \frac{x}{|x|}, \quad \text{(9)} \lim_{x \to \infty} \frac{\ln(\sqrt{x})}{x^2}.$$