Warm-up

Suppose $f(x)$ is a differentiable function. Then in terms of x, $f(x)$ and $f'(x)$, what are

1. $\frac{d}{dx}(xf(x))$
2. $\frac{d}{dx}(f(x)+\sin(x))$
3. $\frac{d}{dx}\left(f(x)/x\right)$
4. $\frac{d}{dx}\left(f^2(x)\right)$
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6. $\frac{d}{dx}\left(\cos\left(xe^{f(x)}\right)\right)$

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4. $\frac{d}{dx}\left(f^2(x)\right) = 2f(x)f'(x)$

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6. $\frac{d}{dx}\left(\cos\left(xe^{f(x)}\right)\right) = -\sin\left(xe^{f(x)}\right) * (1 + xf'(x)) * e^{f(x)}$

Important reminder:

The notes from class are **not comprehensive**!

To be fully prepared for homework and exams, you must also do the **reading from the book**.

The approximate reading schedule for the entire semester, as well as book problems analogous to those on the homework, are posted on the website.

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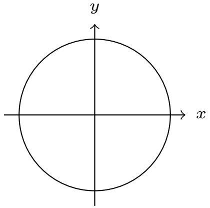

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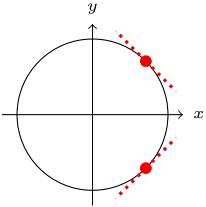

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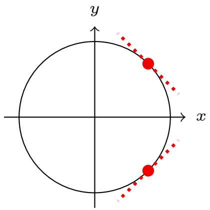

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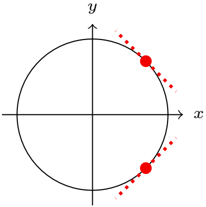

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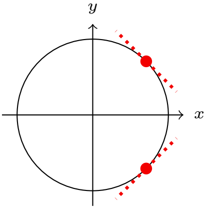

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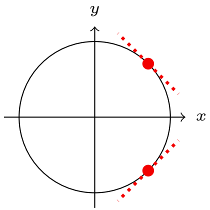

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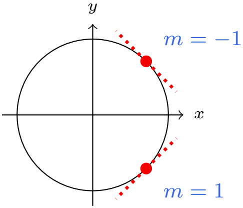

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Examples

Calculate $\frac{dy}{dx}$ if...

(a) $xy = 1$

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(c) $y/x = xy^4 + 1$

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Then $\frac{dx}{dy}$ for (a) and (b) as well. To do this, now pretend that $x = f(y)$. See a pattern?

For reference, the graphs look like

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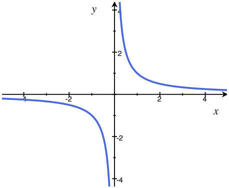

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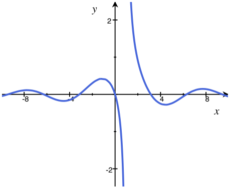

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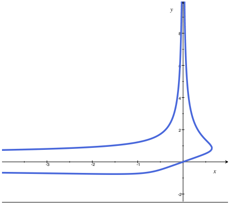

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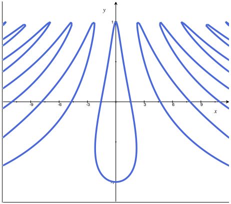

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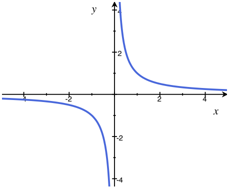

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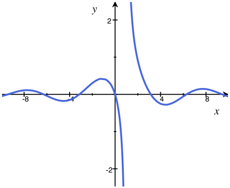

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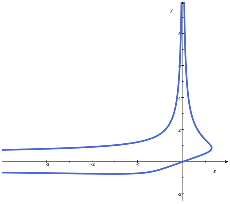

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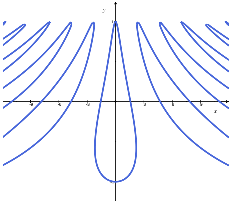

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Vertical and horizontal tangent lines

We saw that if $x^2 + y^2 = 1$, then $\frac{dy}{dx} = -x/y$.

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To get the x-coordinate, plug back in and solve:

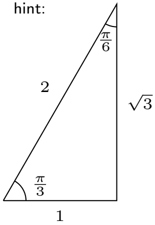
$$x^2 + 0^2 = 1, \quad \text{so } x = \pm 1.$$

Answer: $\boxed{\text{at } (1,0) \text{ and } (-1,0).}$

Other tangent lines

We saw that if $x^2 + y^2 = 1$, then $\frac{dy}{dx} = -x/y$.

Question 3: Find the equations for all lines which are tangent to the unit circle and have slope $\sqrt{3}$.



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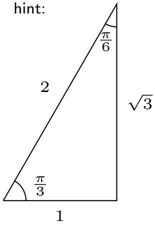

Solution: Solve $\sqrt{3} = \frac{dy}{dx} = -x/y$.

This happens when

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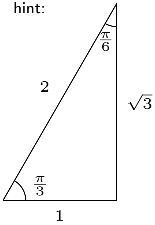

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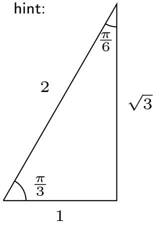

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At $\theta = -\pi/6$, we have $x = \sqrt{3}/2, y = -1/2$, so the tangent line is

$$\boxed{y = \sqrt{3}(x - \sqrt{3}/2) - 1/2}$$

At $\theta = 5\pi/6$, we have $x = -\sqrt{3}/2, y = 1/2$, so the tangent line is

$$\boxed{y = \sqrt{3}(x + \sqrt{3}/2) + 1/2}$$

Normal lines

The line normal to a curve $F(x,y) = c$ at a point (x_0, y_0) is the line that goes through (x_0, y_0) and is perpendicular to the tangent line. *(Recall if $\ell_1 \perp \ell_2$ are perpendicular lines with slopes m_1 and m_2, respectively, then $m_2 = -1/m_1$.)*

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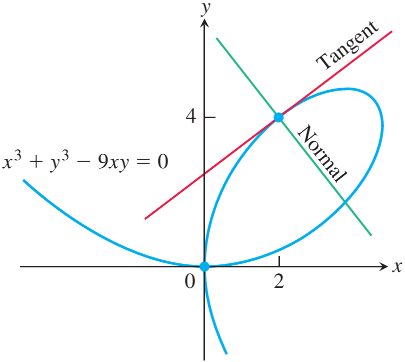
Example: Compute the lines tangent and normal to the curve $x^3 + y^3 - 9xy = 0$ at the point $(2, 4)$.



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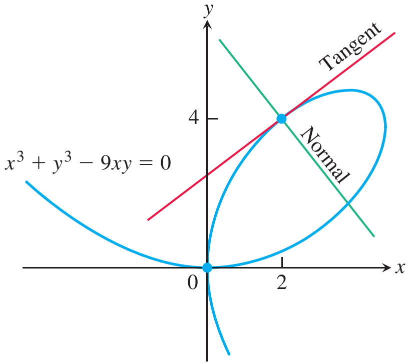

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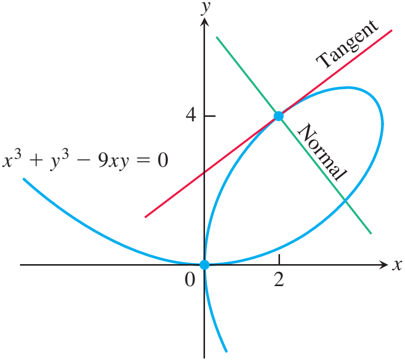


$3x^2 + 3y^2 \frac{dy}{dx} - 9y - 9x \frac{dy}{dx} = 0,$ so $\frac{dy}{dx} = (-3x^2 + 9y)/(3y^2 - 9x)$

Examples

Calculate the lines normal to each curve at the given point....

(a) $xy = 1$
at $(-2, -1/2)$

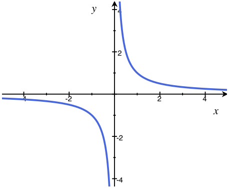
(b) $y + \sin(x) = xy$
at $(0, 0)$

(c) $y/x = xy^4 + 1$
at $(0, 0)$

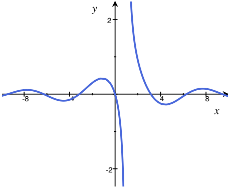
(d) $\cos(xe^y) = y^2$
at $(0, -1)$

For reference, the graphs look like

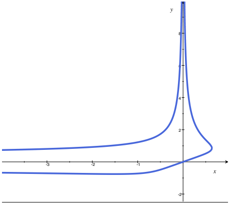
(a)



(b)



(c)



(d)