Warmup

Recall:

$$\frac{d}{dx}e^x = e^x \quad \text{and} \quad \frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

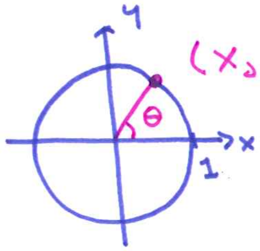
Calculate...

1. $\frac{d}{dx}e^{17x}$

2. $\frac{d}{dx}e^{x\ln(3)}$

3. $\frac{d}{dx}e^{\sqrt{x^2+x}}$

Trig Identities:



$(x, y) = (\cos\theta, \sin\theta)$

Resulting identities:

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\cos^2(\theta) + \sin^2\theta = 1$

Other useful identities:

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

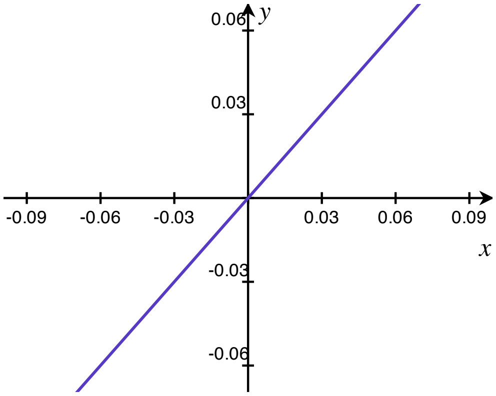
$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$

The derivative of sine

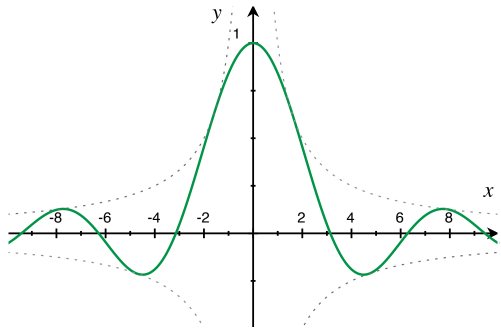
$$\begin{aligned}
\frac{d}{dx}\sin x &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin(x)(\cos(h)-1) + \cos(x)\sin(h)}{h} \\
&= \sin(x)\lim_{h\to 0}\frac{\cos(h)-1}{h} + \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h}
\end{aligned}$$

Recall: $\cos(0) = 1$ and $\sin(0) = 0$

Near $x = 0$, $\sin(x) \approx x$:

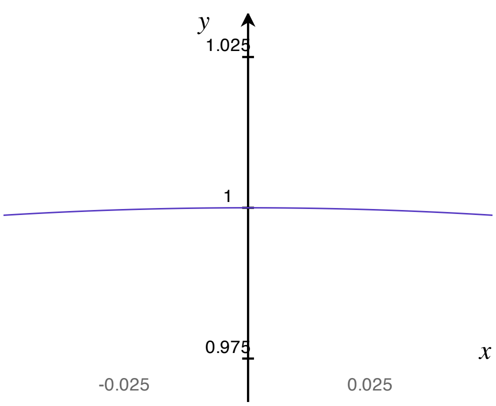


Graph of $\frac{\sin(x)}{x}$:

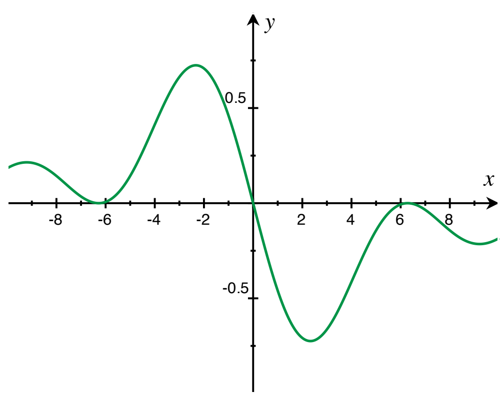


$$\boxed{\lim_{x \to 0} \frac{\sin(x)}{x} = 1}$$

Near $x = 0$, $\cos(x) \approx 1$:



Graph of $\frac{\cos(x)-1}{x}$:



$$\boxed{\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0}$$

The derivative of sine

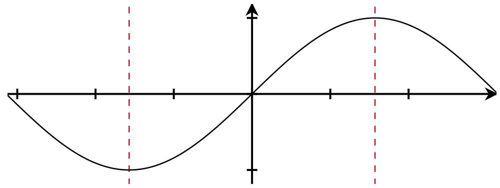
$$\begin{aligned}
\frac{d}{dx}\sin x &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin(x)(\cos(h)-1)+\cos(x)\sin(h)}{h} \\
&= \sin(x)\lim_{h\to 0}\frac{\cos(h)-1}{h}+\cos(x)\lim_{h\to 0}\frac{\sin(h)}{h} \\
&= \sin(x)*0+\cos(x)*1 \\
&= \boxed{\cos(x)}
\end{aligned}$$

The derivative of cosine

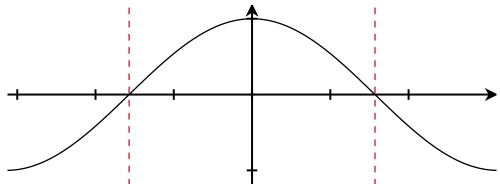
$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h} \\
&= \lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h} \\
&= \lim_{h\to 0}\frac{\cos(x)(\cos(h)-1)-\sin(x)\sin(h)}{h} \\
&= \cos(x)\lim_{h\to 0}\frac{\cos(h)-1}{h}-\sin(x)\lim_{h\to 0}\frac{\sin(h)}{h} \\
&= \cos(x)*0-\sin(x)*1 \\
&= \boxed{-\sin(x)}
\end{aligned}$$

Does it make sense?

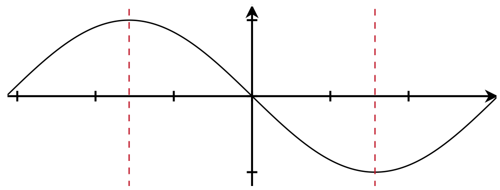
$y = \sin(x)$:



$y = \cos(x)$:



$y = -\sin(x)$:



Examples

Calculate...

1. $\frac{d}{dx}\sin(2x) = \boxed{2 * \cos(2x)}$

2. $\frac{d}{dx}\cos(3x + \sqrt{x})$
   $= \frac{d}{dx}\cos(3x + x^{1/2}) = \boxed{(3 + \frac{1}{2}x^{-\frac{1}{2}})(-\sin(3x + x^{1/2}))}$

3. $\frac{d}{dx}\sin(x)\cos(x) \quad = \sin(x)(-\sin(x)) + \cos(x)\cos(x)$
   $= \boxed{\cos^2(x) - \sin^2(x)}$

   Notice: $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$, and $\cos^2(x) - \sin^2(x) = \cos(2x)$.

   Does your answer still make sense from this perspective?

4. $\frac{d}{dx}\sin(\cos(x^2 + 2)) = \cos(\cos(x^2 + 2)) * \frac{d}{dx}\left(\cos(x^2 + 2)\right)$
   $= \cos(\cos(x^2 + 2)) * (-\sin(x^2 + 2)) * \frac{d}{dx}\left(x^2 + 2\right)$
   $= \boxed{\cos(\cos(x^2 + 2)) * (-\sin(x^2 + 2)) * (2x)}$

On your own, fill in the rest of the trig functions:

1. $\frac{d}{dx} \tan(x)$

2. $\frac{d}{dx} \cot(x)$

3. $\frac{d}{dx} \sec(x)$

4. $\frac{d}{dx} \csc(x)$

Derivative so far

Definition: $f'(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Combining functions:

$$\frac{d}{dx} c * f(x) = c * f'(x) \qquad \frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$

$$\frac{d}{dx}\left(f(x) * g(x)\right) = f(x)g'(x) + g(x)f'(x) \qquad \frac{d}{dx}(f(g(x)) = f'(g(x)) * g'(x)$$

Basic functions:

$f(x)$	x^a	$\sin(x)$	$\cos(x)$	e^x
$f'(x)$	ax^{a-1}	$\cos(x)$	$-\sin(x)$	e^x

Other functions:

$f(x)$	$\tan(x)$	$\cot(x)$	$\sec(x)$	$\csc(x)$	a^x
$f'(x)$	$\sec^2(x)$	$-\csc^2(x)$	$\sec(x)\tan(x)$	$-\csc(x)\cot(x)$	$\ln(a)a^x$

Example

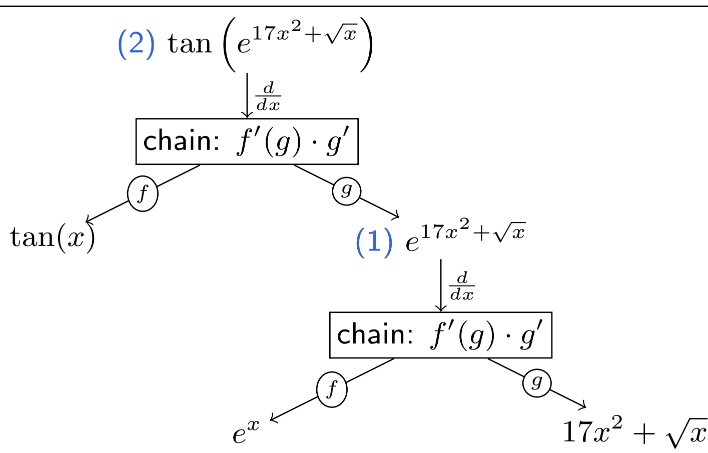
Compute the derivatives of

(1) $e^{17x^2+\sqrt{x}} = e^{17x^2+\sqrt{x}} * \left(17 * 2x + \frac{1}{2\sqrt{x}}\right)$

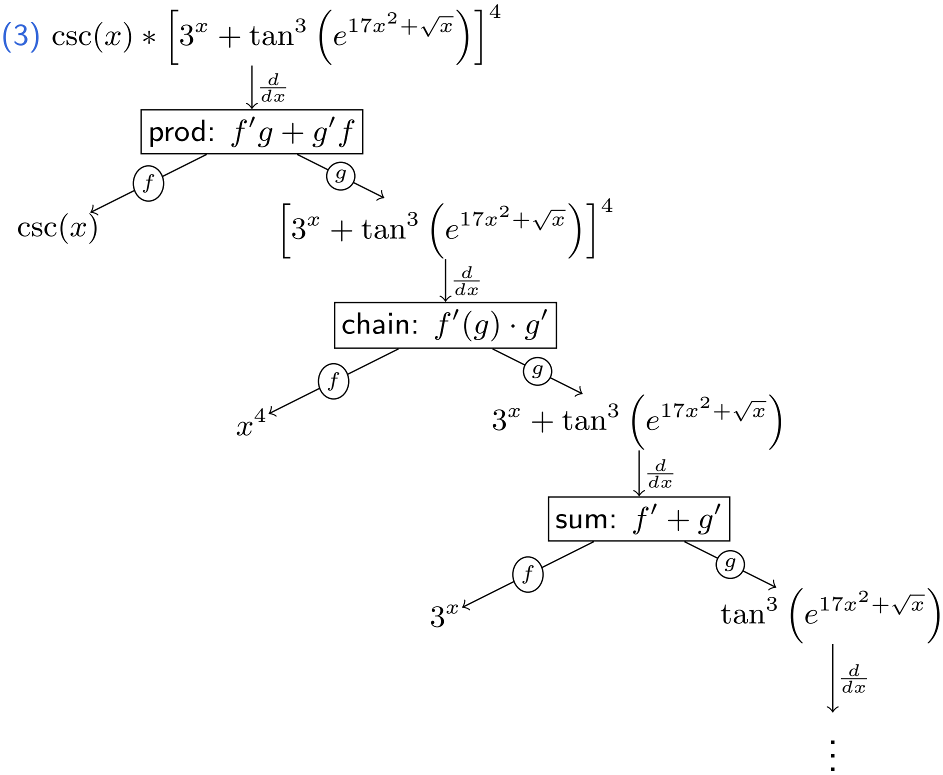
(2) $\tan\left(e^{17x^2+\sqrt{x}}\right)$

$$= \sec^2\left(e^{17x^2+\sqrt{x}}\right) * \left(e^{17x^2+\sqrt{x}} * \left(17 * 2x + \frac{1}{2\sqrt{x}}\right)\right)$$

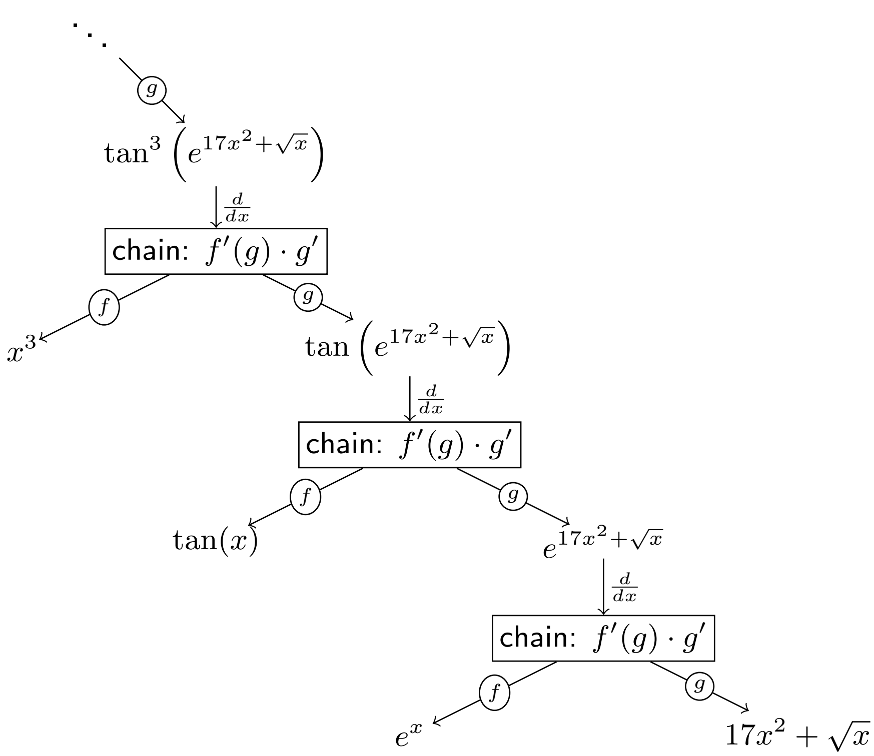
(3) $\csc(x) * \left[3^x + \tan^3\left(e^{17x^2+\sqrt{x}}\right)\right]^4$



(3)



$\csc(x) * \left[3^x + \tan^3\left(e^{17x^2+\sqrt{x}}\right)\right]^4$
$\frac{d}{dx}$
prod: $f'g + g'f$
f
g
$\csc(x)$
$\left[3^x + \tan^3\left(e^{17x^2+\sqrt{x}}\right)\right]^4$
$\frac{d}{dx}$
chain: $f'(g) \cdot g'$
f
g
x^4
$3^x + \tan^3\left(e^{17x^2+\sqrt{x}}\right)$
$\frac{d}{dx}$
sum: $f' + g'$
f
g
3^x
$\tan^3\left(e^{17x^2+\sqrt{x}}\right)$
$\frac{d}{dx}$



g
$\tan^3\left(e^{17x^2+\sqrt{x}}\right)$
$\frac{d}{dx}$
chain: $f'(g) \cdot g'$
f
g
x^3
$\tan\left(e^{17x^2+\sqrt{x}}\right)$
$\frac{d}{dx}$
chain: $f'(g) \cdot g'$
f
g
$\tan(x)$
$e^{17x^2+\sqrt{x}}$
$\frac{d}{dx}$
chain: $f'(g) \cdot g'$
f
g
e^x
$17x^2 + \sqrt{x}$