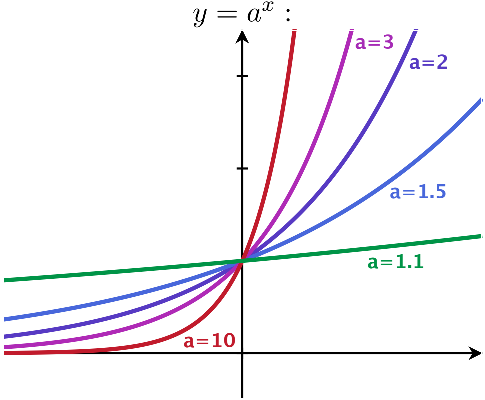
Recall: Our favorite exponential function

Look at how the curve $y = a^x$ is increasing through the point $(0, 1)$:



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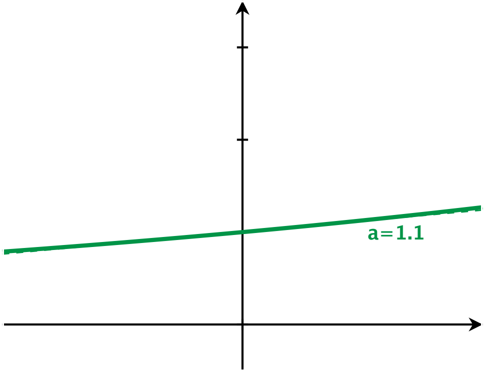

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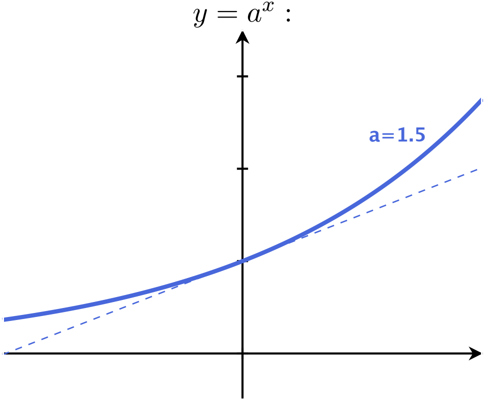

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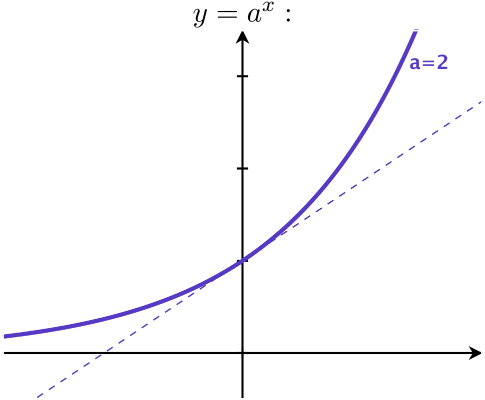

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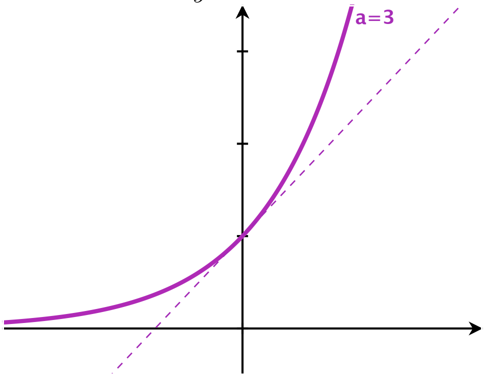

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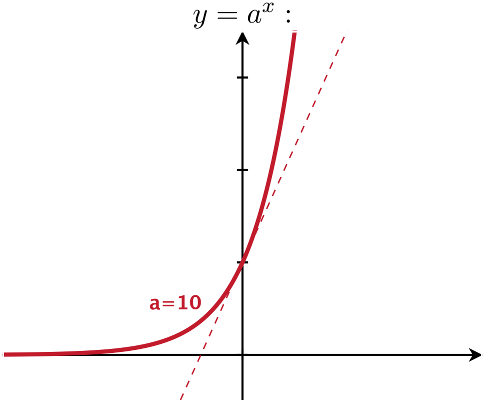

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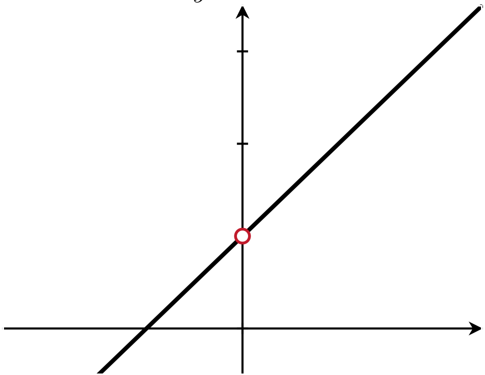

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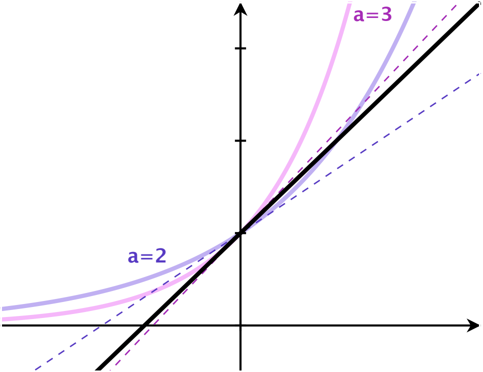

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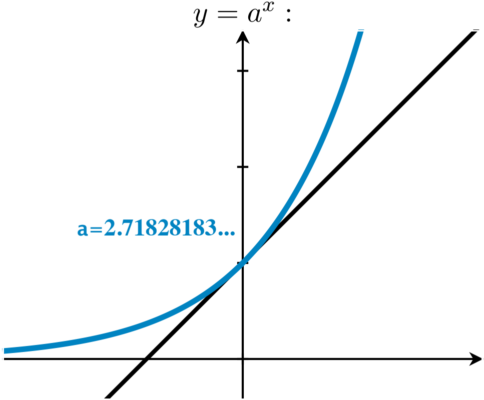

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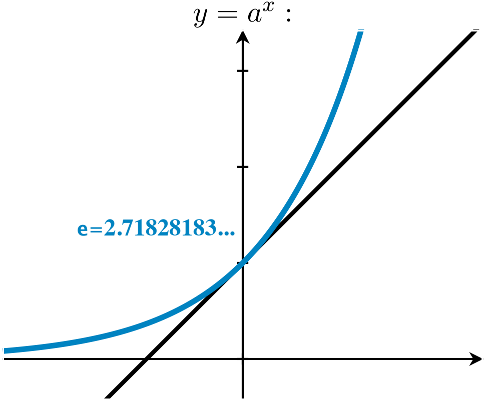


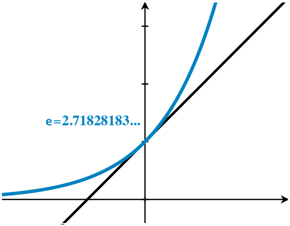
Q: Is there an exponential function whose slope at (0,1) is 1?

A: e^x is the exponential function whose slope at (0,1) is 1.

($e = 2.71828183\ldots$ is to calculus as $\pi = 3.14159265\ldots$ is to geometry)

Derivative of exponential functions

We *defined* e as the number such that the curve $y = e^x$ has slope $m = 1$ at the point $(0, 1)$.



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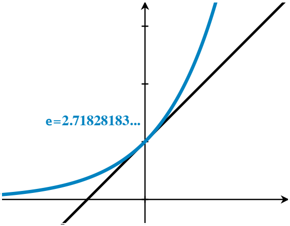

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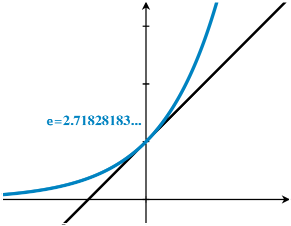

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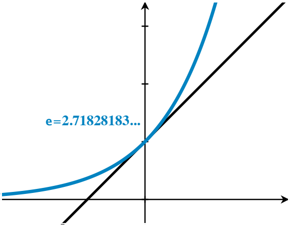

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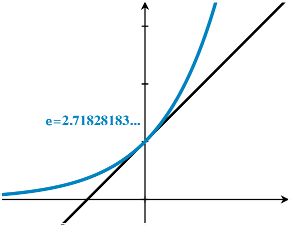

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So we may take for granted that

$$\boxed{\lim_{h \to 0} \frac{e^h - 1}{h} = 1}.$$

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Recall that $\ln(x)$ is the inverse function of e^x, so that

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Recall chain rule: $\boxed{\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)}$.

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Note: $\ln(e) = 1$, so these rules agree when $a = e$. ✓

You try: Compute the derivatives of the following equations.

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2. $f(x) = 3^x$
3. $f(x) = (1/5)^x$
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You try: Compute the derivatives of the following equations.

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6. $f(x) = 3^{4x^2-5x+e^x}$ Ans: $\boxed{3^{4x^2-5x+e^x} \cdot \ln(3) \cdot (8x-5)}$

Some applications of derivatives

Let x_0 be a real number. The **instantaneous rate of change** of $f(x)$ with respect to x, at x_0, is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

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So

$$\left.\frac{d}{dt}A\right|_{t=t_0} = 2\pi \cdot r(t_0) \cdot r'(t_0) = 2\pi \cdot 5 \cdot 7 = \boxed{70\pi}.$$

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Variable: t Function: $A = A(r(t)) = \pi(r(t))^2$

Answer:

$$\frac{d}{dt}A = \frac{d}{dt}A(r(t)) = \underbrace{\frac{dA}{dr} \cdot \frac{dr}{dt}}_{\text{Use Leibnitz notation!}} = (2\pi r(t))r'(t).$$

We don't know what time t_0 we care about, but we do know that *at that (mystery) time*,

$$r(t_0) = 5 \qquad \text{and} \qquad r'(t_0) = 7.$$

So

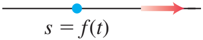
$$\left.\frac{d}{dt}A\right|_{t=t_0} = 2\pi \cdot r(t_0) \cdot r'(t_0) = 2\pi \cdot 5 \cdot 7 = \boxed{70\pi}.$$

Look up "related rates" online.

Motion Along a Line

Let an object move (in time) back and forth along a line according to the function

$$s = f(t).$$



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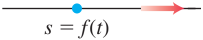
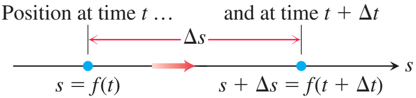


The **displacement** of the object over the time interval from t to $t + \Delta t$ is

$$\Delta s = f(t + \Delta t) - f(t).$$



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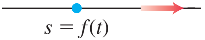

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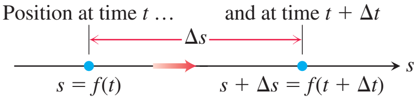
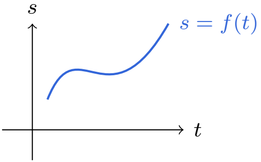


Of course, we could graph s versus t to get a 2-d picture, but the object is still just moving in 1 dimension...



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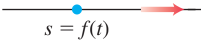

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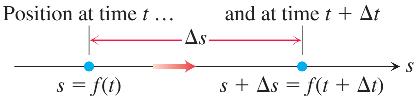

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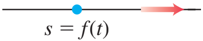

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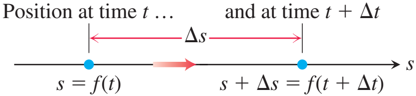

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(Think: Δt is just like h from before!)

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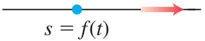

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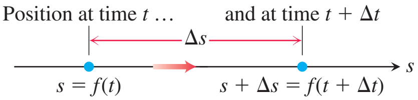

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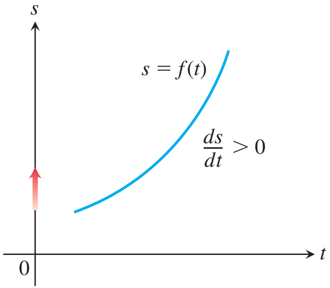
(Think: Δt is just like h from before!)

The **velocity** of the object as a function of time is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Speed versus velocity

If the object is moving *forward*, the velocity is positive.



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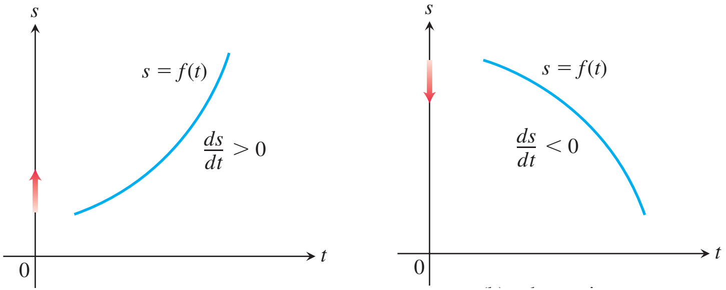

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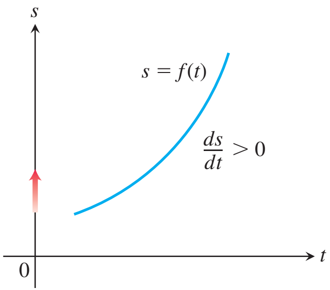

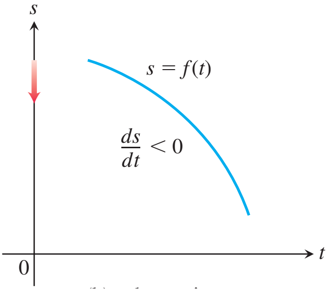

Similarly, if the object is moving *backwards*, the velocity is negative.

The **speed** of the object is the absolute value of velocity,

$$\text{speed} = |v(t)| = \left|\frac{ds}{dt}\right|.$$

Change in velocity

Acceleration $a(t)$ is the change in velocity over time. Namely, it is the derivative of velocity with respect to time:

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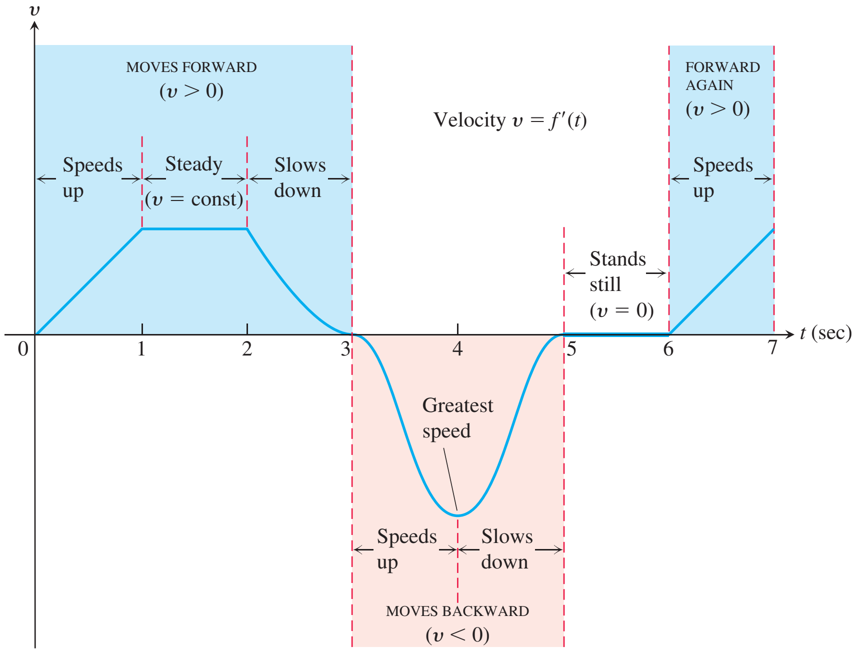
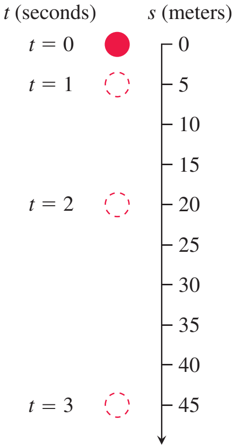


v
MOVES FORWARD
($v > 0$)
Velocity $v = f'(t)$
FORWARD
AGAIN
($v > 0$)
Speeds
up
Steady
(v = const)
Slows
down
Speeds
up
Stands
still
($v = 0$)
0
1
2
3
4
5
6
7
t (sec)
Greatest
speed
Speeds
up
Slows
down
MOVES BACKWARD
($v < 0$)

Acceleration under gravity



Near the surface of Earth all bodies fall with the same constant acceleration:

$$s = \frac{1}{2}gt^2, \quad \text{where} \quad g = 9.8\text{m/s}^2.$$

(Frame of reference: positive is down.)

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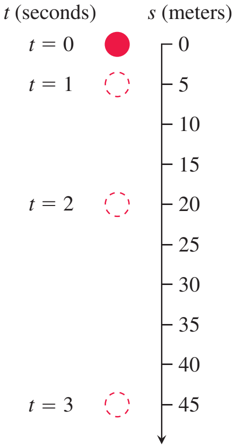

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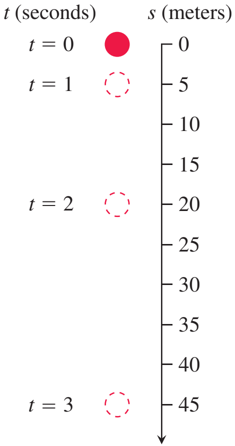

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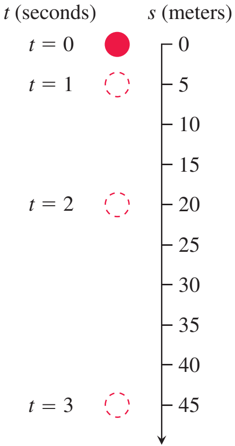

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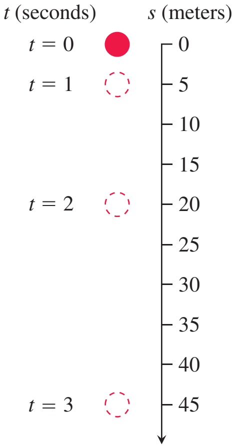

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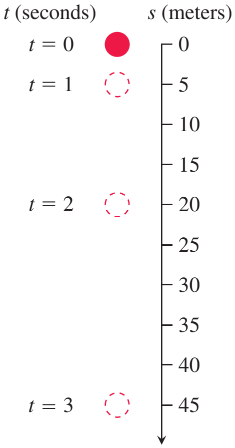

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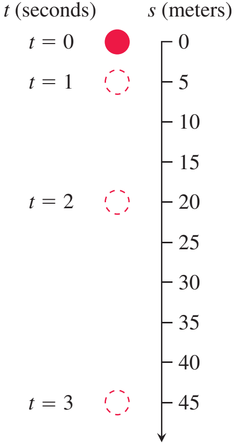

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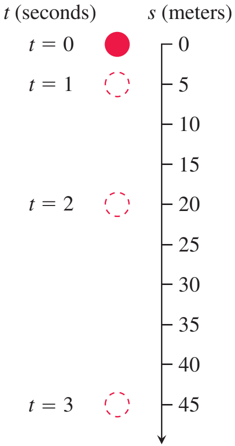

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Example: We drop a ball from a very high tower. How far has it fallen after 10 seconds? How fast is it going at that point?

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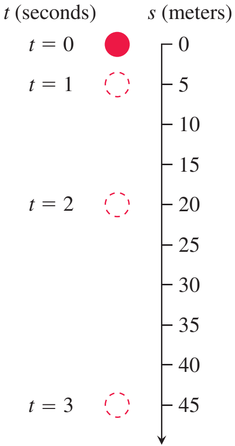

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Ans.

$$s(10) = \frac{1}{2} \cdot g \cdot (10)^2 = 980/2 \text{ meters}, \quad v(10) = g(t) = 98\text{m}/\text{s}.$$

Acceleration under gravity

Near the surface of Earth, the vertical trajectory of a body is given by

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0,$$

where

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Use $\boxed{s(t) = \frac{1}{2}(-9.8)t^2 + 400t + 1}$.

Rates of change in economics

Let's consider the cost of production $c(x)$ as a function of x, the number of units produced. (The first thing is expensive to make; manufacturing in bulk can be more efficient.)

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Read examples 5 and 6 in the book.

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See also "sensitivity to change", e.g. with genetic data. (Example 7)

Review

- Functions, basic graphs, graph transformations
- Domains and ranges
- Trig functions and identities, inverse trig functions
- Exponential functions, logarithms, and identities
- Limits
  - one- and two-sided
  - when are they defined
  - computing them
- Asymptotes
- Continuity
- Average rate of change
- Limit definition of derivatives
  - polynomials, roots, reciprocals
- Basic derivative rules
  - powers, scalars, sums, products, compositions

Functions, basic graphs, graph transformations

Know basic graphs of

$$mx + b, \quad x^2, \quad x^3, \quad x^4.$$
$$1/x, \quad 1/x^2, \quad \sqrt{x}, \quad \sqrt[3]{x}.$$

If you know the graph of $y = f(x)$, also know the graphs of

$$f(x + c), \quad f(cx), \quad cf(x), \quad f(x) + c, \quad 1/f(x).$$

Also know how graph transformations affect domain and range.

Trig functions and identities, inverse trig functions

- Graphs of $\sin(x)$ and $\cos(x)$
- How to use the unit circle
- Special values
- Angle addition formulas
- How to compute $\tan(x), \csc(x), \sec(x), \cot(x)$ and their graphs

Exponential functions, logarithms, and identities

- Graphs of a^x for $a > 0$ and for $a < 0$
- What is e?
- Identities like $a^{x+y} = a^x a^y$, etc.
- Graphs of $\log_a(x)$. What is $\ln(x)$?
- Identities like $\ln(xy) = \ln(x) + \ln(y)$.
- Exponential growth.

Limits and continuity

- One sided limits, from the left or right
- Two-sided limits
- Limits at $\pm\infty$
- Infinite limits
- Computing standard limits
- Graph asymptotes (vertical, horizontal, skew).
- Definition of continuous, and how to compute where functions are discontinuous.
  (Difference between the limit existing and a function being continuous.)

Rates of change

- Average rate of change
$$\frac{f(x+h)-f(x)}{h}$$
- Limit definition of derivative
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$
- Computing derivatives of functions like $mx+b, x^2, x^3, \sqrt{x}, 1/x, 1/x^2$ using the limit definition.
- Basic derivative rules: powers, scalars, sums, products, compositions