Today: Derivatives of a function

Warmup:

1. Let $f(x) = \sqrt{x}$. Compute the following limits.
   (a) $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$
   (b) $\lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$
   (c) $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, where a is non-negative real number.
2. Let $f(x) = x^2 + 3x$. Compute the following limits.
   (a) $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$
   (b) $\lim_{h \to 0} \frac{f(-5+h) - f(-5)}{h}$
   (c) $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, where a is non-negative real number.

Answers: 1(a) $1/2$, (b) $1/2\sqrt{5}$, (c) $1/2\sqrt{a}$; 2(a) 9, (b) -7, (c) $2a + 3$.

Definition

The **derivative** of a function f is a new function defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

We will say that a function f is **differentiable** over an interval (a, b) if the derivative function $f'(x)$ exists at every point in (a, b).

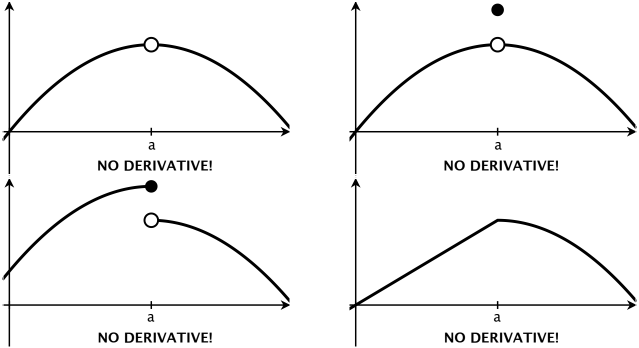
Definition

The **derivative** of a function f is a new function defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

We will say that a function f is **differentiable** over an interval (a, b) if the derivative function $f'(x)$ exists at every point in (a, b).

Remember from last time:



Definition

The **derivative** of a function f is a new function defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

We will say that a function f is **differentiable** over an interval (a, b) if the derivative function $f'(x)$ exists at every point in (a, b).

(see notes from last time for when derivatives *don't* exist)

Q. How is this the same or different from what we were doing last time with tangent lines?

Definition

The **derivative** of a function f is a new function defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

We will say that a function f is **differentiable** over an interval (a, b) if the derivative function $f'(x)$ exists at every point in (a, b).

(see notes from last time for when derivatives *don't* exist)

Q. How is this the same or different from what we were doing last time with tangent lines?

A. Last time, we were calculating derivatives at individual points, and getting **numbers** for answers. Today, we'll calculate the **derivative function**, and get out answers with variables in them (do all the points at once).

Example: let $f(x) = x^2$

Derivatives at a point: If I first ask "what is $f'(2)$?", I could calculate

$$f'(2) = \lim_{h\to 0} \frac{(2+h)^2 - 2^2}{h} = \lim_{h\to 0} \frac{4h + h^2}{h} = \lim_{h\to 0} 4 + h = \boxed{4}.$$

Example: let $f(x) = x^2$

Derivatives at a point: If I first ask "what is $f'(2)$?", I could calculate

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = \lim_{h \to 0} \frac{4h + h^2}{h} = \lim_{h \to 0} 4 + h = \boxed{4}.$$

But then, if I ask "what is $f'(3)$?" we have to do it all over again.

Example: let $f(x) = x^2$

Derivatives at a point: If I first ask "what is $f'(2)$?", I could calculate

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = \lim_{h \to 0} \frac{4h + h^2}{h} = \lim_{h \to 0} 4 + h = \boxed{4}.$$

But then, if I ask "what is $f'(3)$?" we have to do it all over again.

Today's goal: Write down a **function** $f'(x)$ which has all the derivatives-at-a-point collected together.

If a is a number, (like 2 or 3) then

$$f'(a) = \underbrace{\lim_{h \to 0}}_{\text{gets rid of the } h\text{'s}} \underbrace{\frac{f(a+h) - f(a)}{h}}_{\text{a function of } a\text{'s and } h\text{'s}}$$

If a is a number, (like 2 or 3) then

$$f'(a) = \underbrace{\lim_{h \to 0}}_{\text{gets rid of the } h\text{'s}} \underbrace{\frac{f(a+h) - f(a)}{h}}_{\text{a function of } a\text{'s and } h\text{'s}} = \text{number}$$

If a is a number, (like 2 or 3) then

$$f'(a) = \underbrace{\lim_{h \to 0}}_{\text{gets rid of the } h\text{'s}} \underbrace{\frac{f(a+h) - f(a)}{h}}_{\text{a function of } a\text{'s and } h\text{'s}} = \text{number}$$

But x is a variable, so

$$f'(x) = \underbrace{\lim_{h \to 0}}_{\text{gets rid of the } h\text{'s}} \underbrace{\frac{f(x+h) - f(x)}{h}}_{\text{a function of } x\text{'s and } h\text{'s}}$$

If a is a number, (like 2 or 3) then

$$f'(a) = \underbrace{\lim_{h\to 0}}_{\text{gets rid of the } h\text{'s}} \underbrace{\frac{f(a+h)-f(a)}{h}}_{\text{a function of } a\text{'s and } h\text{'s}} = \text{number}$$

But x is a variable, so

$$f'(x) = \underbrace{\lim_{h\to 0}}_{\text{gets rid of the } h\text{'s}} \underbrace{\frac{f(x+h)-f(x)}{h}}_{\text{a function of } x\text{'s and } h\text{'s}} = \text{function of } x\text{'s}$$

Starting simple

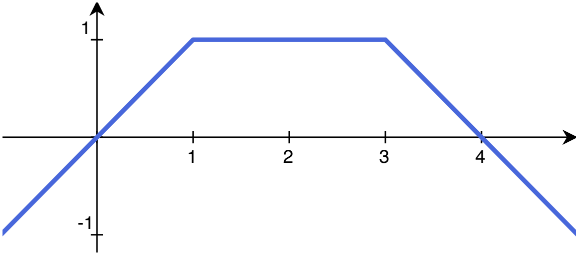
Suppose we consider the piecewise linear function

$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$

Starting simple

Suppose we consider the piecewise linear function

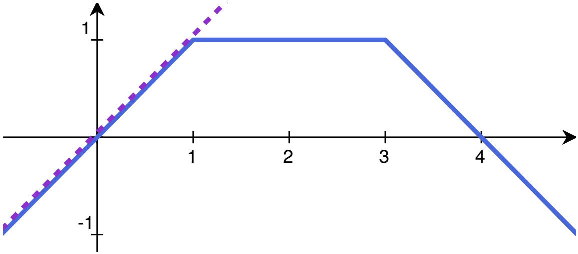
$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$



Starting simple

Suppose we consider the piecewise linear function

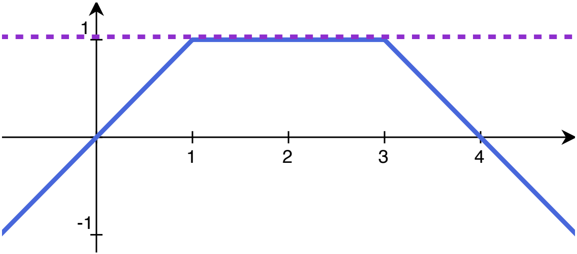
$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$



Starting simple

Suppose we consider the piecewise linear function

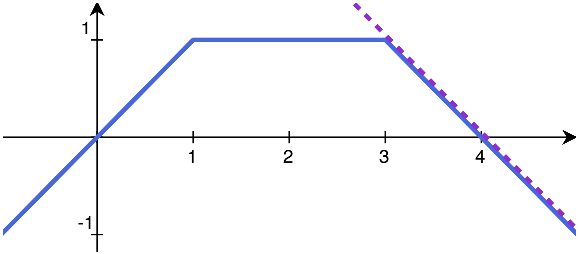
$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$



Starting simple

Suppose we consider the piecewise linear function

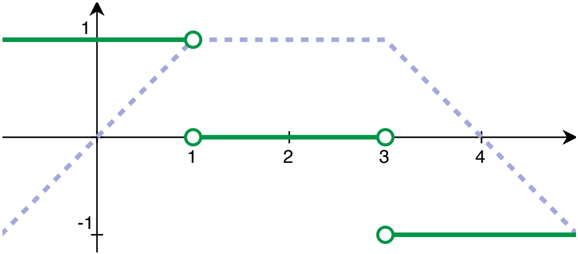
$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$



Starting simple

Suppose we consider the piecewise linear function

$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$



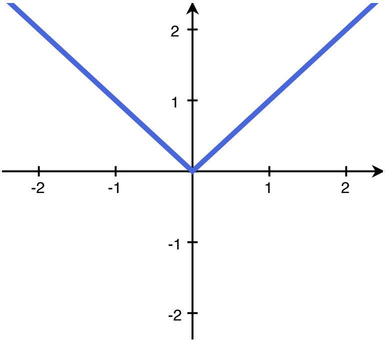
The derivative is:

$$f'(x) = \begin{cases} 1 & x < 1 \\ 0 & 1 < x < 3 \\ -1 & 3 < x \end{cases}$$

Another example

What is the derivative of $f(x) = |x|$?

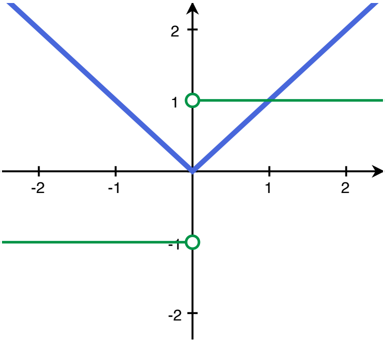
Write down the piecewise function and sketch it on the graph.



Another example

What is the derivative of $f(x) = |x|$?

Write down the piecewise function and sketch it on the graph.

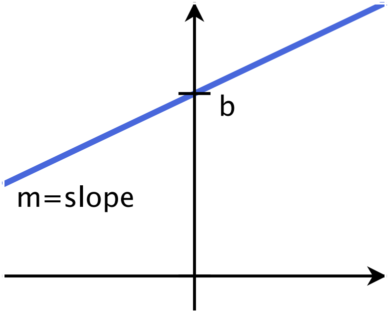


$$f'(x) = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$$

Lines

In general, if m and b are constants, and

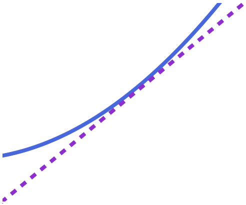
$$f(x) = mx + b$$



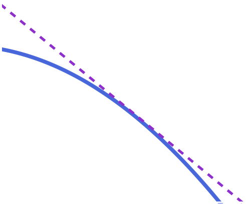
$$f'(x) = m$$

the slope of the tangent line = slope of the line

Rough shape of the derivative



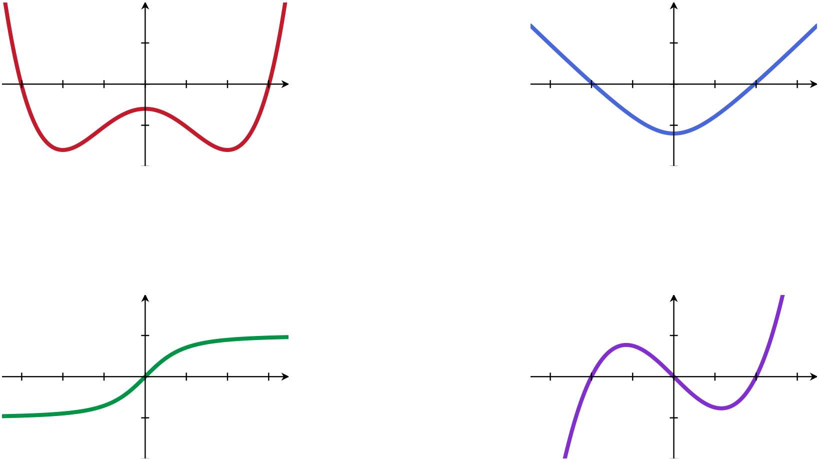
if $f(x)$ is increasing
$f'(x)$ is positive!



if $f(x)$ is decreasing
$f'(x)$ is negative!

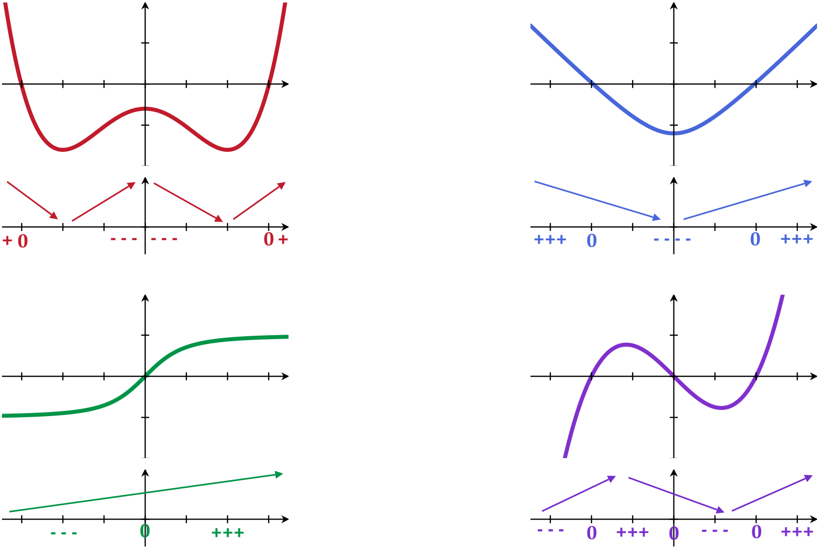
Match em up!

Here are graphs of two functions and their derivatives. Which are which?



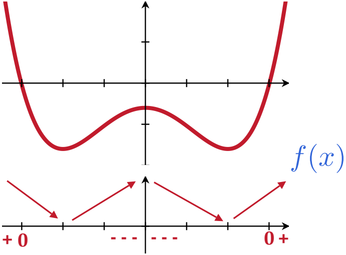
Match em up!

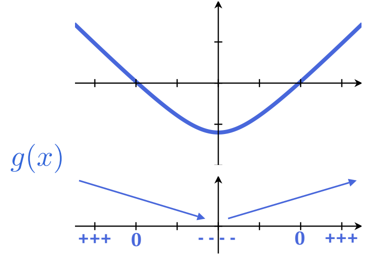
Here are graphs of two functions and their derivatives. Which are which?

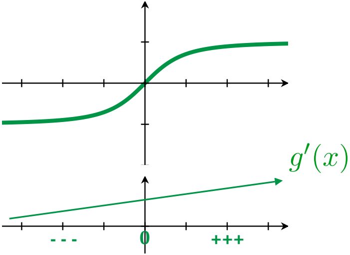


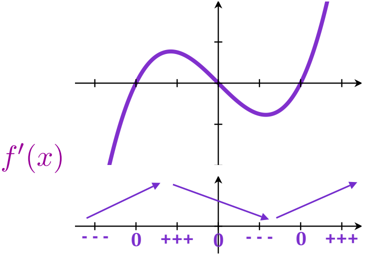
Match em up!

Here are graphs of two functions and their derivatives. Which are which?









A little more notation

Back to what $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ means:

A little more notation

Back to what $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h)-f(x) = \Delta y}$

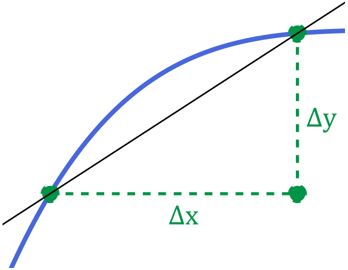
(Δ means "change")

A little more notation

Back to what $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



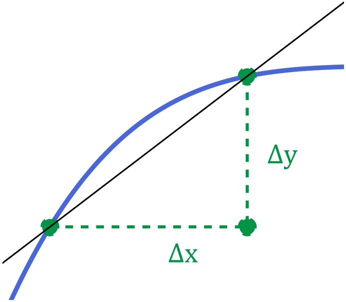
So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



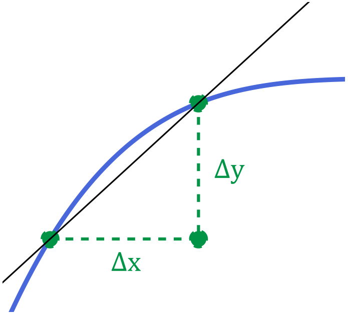
So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



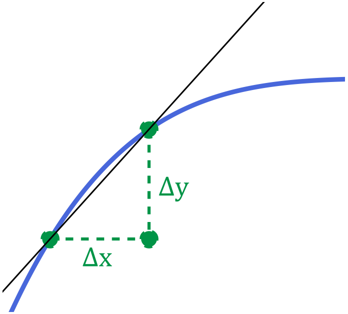
So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



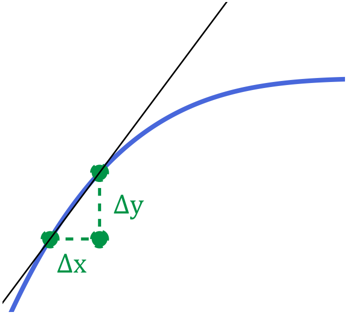
So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



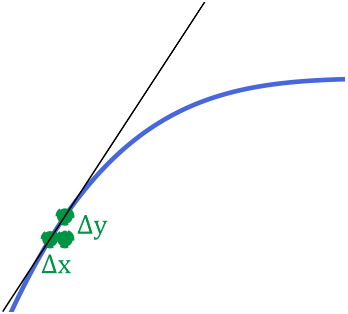
So $m = \frac{f(x+h) - f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



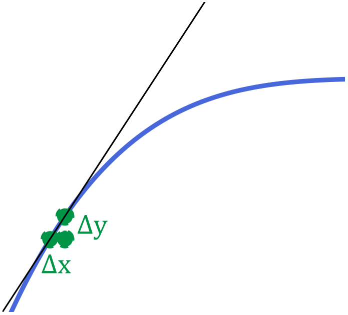
So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



As $h \to 0$,
Δx and Δy get infinitely small.

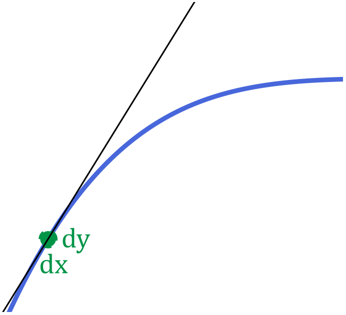
So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

A little more notation

Back to what $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ means:

Rename $\boxed{h = \Delta x}$ and $\boxed{f(x+h) - f(x) = \Delta y}$

(Δ means "change")



As $h \to 0$,
Δx and Δy get infinitely small.

$$\Delta x \rightsquigarrow dx \qquad \Delta y \rightsquigarrow dy$$

$$\frac{\Delta y}{\Delta x} \rightsquigarrow \frac{dy}{dx}$$

"infinitesimals"

So $m = \frac{f(x+h)-f(x)}{h} = \frac{\Delta y}{\Delta x}$.

Leibniz notation

One way to write the derivative of $f(x)$ versus x is $f'(x)$.
Another way to write it is

$$f'(x) = \frac{df}{dx} = \frac{d}{dx}f(x).$$

Leibniz notation

One way to write the derivative of $f(x)$ versus x is $f'(x)$. Another way to write it is

$$f'(x) = \frac{df}{dx} = \frac{d}{dx}f(x).$$

Derivatives at a point: $f'(a)$ means the derivative of $f(x)$ **evaluated at a**. Another way to write it is

$$f'(a) = \left.\frac{df}{dx}\right|_{x=a} = \left.\frac{d}{dx}f(x)\right|_{x=a}$$

Leibniz notation

One way to write the derivative of $f(x)$ versus x is $f'(x)$.
Another way to write it is

$$f'(x) = \frac{df}{dx} = \frac{d}{dx}f(x).$$

Derivatives at a point: $f'(a)$ means the derivative of $f(x)$ **evaluated at a**. Another way to write it is

$$f'(a) = \left.\frac{df}{dx}\right|_{x=a} = \left.\frac{d}{dx}f(x)\right|_{x=a}$$

Example: We can write the derivative of x^2 as $\frac{d}{dx}x^2$

and the derivative of x^2 at $x = 5$ as $\left.\frac{d}{dx}x^2\right|_{x=5}$

Leibniz notation

One way to write the derivative of $f(x)$ versus x is $f'(x)$.
Another way to write it is

$$f'(x) = \frac{df}{dx} = \frac{d}{dx} f(x).$$

Derivatives at a point: $f'(a)$ means the derivative of $f(x)$ **evaluated at a**. Another way to write it is

$$f'(a) = \left.\frac{df}{dx}\right|_{x=a} = \left.\frac{d}{dx} f(x)\right|_{x=a}$$

Example: We can write the derivative of x^2 as $\frac{d}{dx}x^2$

and the derivative of x^2 at $x = 5$ as $\left.\frac{d}{dx}x^2\right|_{x=5}$

Other notation:

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = D(f)(x) = D_x f(x).$$

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$
$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$

$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h}$$

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$

$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} 2x+h$$

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$

$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} 2x+h = \boxed{2x}$$

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\begin{aligned}
\frac{d}{dx}x^2 &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h} \\
&= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h} \\
&= \lim_{h\to 0} 2x+h = \boxed{2x}
\end{aligned}$$

(so $\left.\frac{d}{dx}x^2\right|_{x=5} = 2*5$)

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$
$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h}$$
$$= \lim_{h\to 0} 2x+h = \boxed{2x} \qquad \text{(so } \left.\frac{d}{dx}x^2\right|_{x=5} = 2*5\text{)}$$

By taking limits, fill in the rest of the table:

$f(x)$	1	x	x^2	x^3	$\frac{1}{x}$	$\frac{1}{x^2}$	$\sqrt{x}$	$\sqrt[3]{x}$
$f'(x)$			$2x$					

Hints: For $\frac{1}{x^2}$, find a common denominator, and then expand.

For $\sqrt[3]{x}$, try multiplying and dividing by $(\sqrt[3]{x+h})^2 + (\sqrt[3]{x+h})(\sqrt[3]{x}) + (\sqrt[3]{x})^2$.

Go to work: building our first derivative rule.

Example 1: What *is* the derivative of x^2?

$$\frac{d}{dx}x^2 = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$
$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h}$$
$$= \lim_{h\to 0} 2x+h = \boxed{2x} \qquad \text{(so } \left.\frac{d}{dx}x^2\right|_{x=5} = 2*5\text{)}$$

By taking limits, fill in the rest of the table:

$f(x)$	1	x	x^2	x^3	$\frac{1}{x}$	$\frac{1}{x^2}$	$\sqrt{x}$	$\sqrt[3]{x}$
$f'(x)$	0	1	$2x$	$3x^2$	$-\frac{1}{x^2}$	$-\frac{2}{x^3}$	$\frac{1}{2\sqrt{x}}$	$\frac{1}{3(\sqrt[3]{x})^2}$

Hints: For $\frac{1}{x^2}$, find a common denominator, and then expand.

For $\sqrt[3]{x}$, try multiplying and dividing by $(\sqrt[3]{x+h})^2 + (\sqrt[3]{x+h})(\sqrt[3]{x}) + (\sqrt[3]{x})^2$.

$f(x)$	$f'(x)$
$x^0 = 1$	0
$x^1 = x$	1
x^2	$2x$
x^3	$3x^2$
$\frac{1}{x} = x^{-1}$	$-x^{-2}$
$\frac{1}{x^2} = x^{-2}$	$-2x^{-3}$
$\sqrt{x} = x^{1/2}$	$(1/2)x^{-1/2}$
$\sqrt[3]{x} = x^{1/3}$	$(1/3)x^{-2/3}$

$f(x)$	$f'(x)$
$x^0 = 1$	0
$x^1 = x$	1
x^2	$2x$
x^3	$3x^2$
$\frac{1}{x} = x^{-1}$	$-x^{-2}$
$\frac{1}{x^2} = x^{-2}$	$-2x^{-3}$
$\sqrt{x} = x^{1/2}$	$(1/2)x^{-1/2}$
$\sqrt[3]{x} = x^{1/3}$	$(1/3)x^{-2/3}$

Power rule:

$$\frac{d}{dx}x^a = ax^{a-1}$$

Use the power rule to take consecutive derivatives of $x^{5/2}$:

$$x^{5/2} \xrightarrow{\frac{d}{dx}} \boxed{}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{}$$

Use the power rule to take consecutive derivatives of $x^{5/2}$:

$$x^{5/2} \xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2}x^{3/2}}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{}$$

Use the power rule to take consecutive derivatives of $x^{5/2}$:

$$x^{5/2} \xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2}x^{3/2}}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2}x^{1/2}}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2}x^{-1/2}}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2} * \left(-\frac{1}{2}\right) x^{-3/2}}$$

Use the power rule to take consecutive derivatives of $x^{5/2}$:

$$x^{5/2} \xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2}x^{3/2}} \text{ 1st derivative}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2}x^{1/2}} \text{ 2nd derivative}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2}x^{-1/2}} \text{ 3rd derivative}$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2} * \left(-\frac{1}{2}\right) x^{-3/2}}$$

4th derivative

Use the power rule to take consecutive derivatives of $x^{5/2}$:

$$x^{5/2} \xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2}x^{3/2}} \quad \textit{1}^{st} \textit{ derivative} = f'(x) = \frac{d}{dx}x^2$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2}x^{1/2}} \quad \textit{2}^{nd} \textit{ derivative} = f''(x) = \frac{d^2}{dx^2}x^2$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2}x^{-1/2}} \quad \textit{3}^{rd} \textit{ derivative} = f^{(3)}(x) = \frac{d^3}{dx^3}x^2$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2} * \left(-\frac{1}{2}\right) x^{-3/2}}$$

$$\textit{4}^{th} \textit{ derivative} = f^{(4)}(x) = \frac{d^4}{dx^4}x^2$$

Use the power rule to take consecutive derivatives of $x^{5/2}$:

$$x^{5/2} \xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2}x^{3/2}} \quad 1^{st} \text{ derivative} = f'(x) = \frac{d}{dx}x^2$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2}x^{1/2}} \quad 2^{nd} \text{ derivative} = f''(x) = \frac{d^2}{dx^2}x^2$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2}x^{-1/2}} \quad 3^{rd} \text{ derivative} = f^{(3)}(x) = \frac{d^3}{dx^3}x^2$$

$$\xrightarrow{\frac{d}{dx}} \boxed{\frac{5}{2} * \frac{3}{2} * \frac{1}{2} * \left(-\frac{1}{2}\right) x^{-3/2}}$$

$$4^{th} \text{ derivative} = f^{(4)}(x) = \frac{d^4}{dx^4}x^2$$

The n^{th} derivative of $f(x)$ is denoted

$$\underbrace{\frac{d}{dx}\frac{d}{dx}\cdots\frac{d}{dx}}_{n} f(x) = \frac{d^n}{dx^n}f(x) = f^{(n)}(x).$$