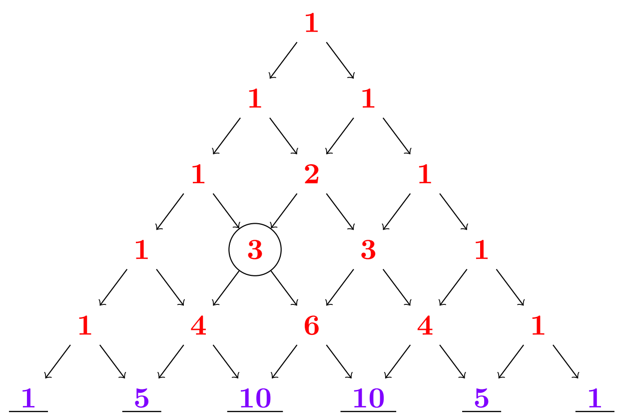
Warmup – quick expansions

Pascal's triangle:



To build Pascal's triangle:

- Start with 1's on the end.
- Add two numbers above to get a new entry.

For example, the circled **3** is the sum of the 1 and 2 above.

(A) Fill in the missing numbers.
Check: your answers should be symmetric (palindromic).

Pascal's triangle is useful for us because its makes expanding things like $(a+b)^n$ quickly–it gets its coefficients from the n^{th} row (where you start counting from 0). For example:

$$(a+b)^0 = \mathbf{1}$$
$$(a+b)^1 = \mathbf{1} * a + \mathbf{1} * b$$
$$(a+b)^2 = \mathbf{1} * a^2 + \mathbf{2} * ab + \mathbf{1} * b^2$$
$$(a+b)^3 = \mathbf{1} * a^3 + \mathbf{3} * a^2b + \mathbf{3} * ab^2 + \mathbf{1} * b^3$$
$$(a+b)^4 = \mathbf{1} * a^4 + \mathbf{4} * a^3b + \mathbf{6} * a^2b^2 + \mathbf{4} * ab^3 + \mathbf{1} * b^4$$
$$(a+b)^5 = \mathbf{1} * a^5 + \mathbf{5} * a^4b + \mathbf{10} * a^3b^2 + \mathbf{10} * a^2b^3 + \mathbf{5} * ab^4 + \mathbf{1} * b^5$$

(B) Fill in the last expansion. Be organized–start with a^5, and lower the a exponent and raise the b exponent each by 1 for each subsequent term. Read the coefficients off the the 5th row you computed above.

Now we can use this to do other kinds of expansions quickly too. For example,

$$\begin{aligned}(h+2)^3 &= \mathbf{1} * h^3 + \mathbf{3} * h^2 * 2 + \mathbf{3} * h * 2^2 + \mathbf{1} * 2^3 \\ &= h^3 + 6h^2 + 12h + 8 \\ (h-3)^4 &= \mathbf{1} * h^4 + \mathbf{4} * h^3(-3) + \mathbf{6} * h^2(-3)^2 + \mathbf{4} * h(-3)^3 + \mathbf{1} * (-3)^4 \\ &= h^4 - 12h^3 + 54h^2 - 108h + 81 \\ (3h-7)^2 &= \mathbf{1} * (3h)^2 + \mathbf{2} * (3h) * (-7) + \mathbf{1} * (-7)^2 \\ &= 9h^2 - 42h + 49\end{aligned}$$

(C) Now you try:

$(h-1)^5 = \boxed{h^5 - 5h^4 + 10h^3 - 10h^2 + 5h - 1}$

$(h+2)^4 = h^4 + 4 * 2 * h^3 + 6 * 2^2 * h^2 + 4 * 2^3 * h + 2^4 = \boxed{h^4 + 8h^3 + 24h^2 + 32h + 16}$

$(2h-1)^3 = (2h)^3 + 3 * (2h)^2 * (-1) + 3 * (2h) * (-1)^2 + (-1)^3 = \boxed{8h^3 - 12h^2 + 6h - 1}$